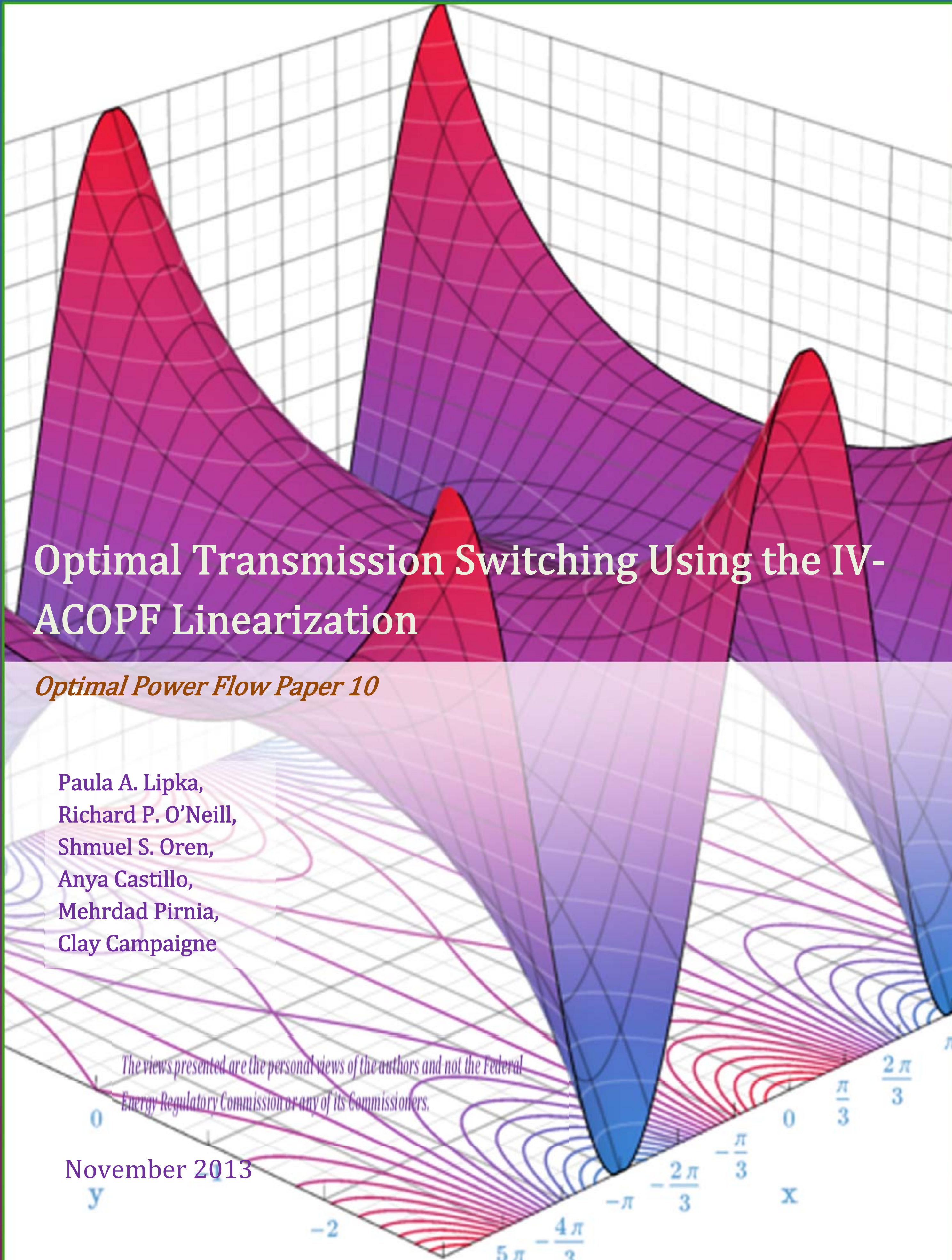

Optimal Transmission Switching Using the IV-ACOPF Linearization

Optimal Power Flow Paper 10

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Abstract

In this paper, we seek to investigate the performance of transmission switching using the iterative linear program approximation to the Current Voltage AC Optimal Power Flow (IV-ACOPF). Several different methods of using this switching are investigated to find a method that addresses the MIP challenges and is both fast and accurate. We consider opening only one line, opening up to five lines, and progressively opening one line at a time. The linear method with switching is much faster than the nonlinear ACOPF and generally finds solutions within 1% of the nonlinear ACOPF.

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1. Introduction

In a given electric power network, some transmission lines are often out of service due to outages or maintenance. However, of the lines that remain operable, removing additional lines may reduce the total power generation cost. Power companies use this practice in times of low load, but often use experience and intuition with reliability testing; that is, without explicitly searching for the optimal topology. The benefit of transmission switching has been demonstrated by Fisher et. al. (2008) and Hedman et. al. (2008) by solving a 'DC' approximation of the alternating current optimal power (ACOPF) flow system. Hedman et. al. (2008) achieved savings of over 20% by switching lines off versus the original network on the IEEE 118 bus test case. Even when the considering network contingencies, transmission switching (again using the 'DC' approximation) can still provide substantial benefits. Hedman et. al. (2008) show that opening five lines reduces costs by 8% in the N-1 compliant RTS-96 system. Potluri and Hedman (2012) suggest that switching has significant benefits over maintaining a static network when solving the full ACOPF. They also demonstrate that an improvement in the system cost using the DC solution procedure does not always give a lower cost in the original AC system. In fact, the DC solution with line switching is AC infeasible or increases the system cost over no switching in several instances.

The nonlinear ACOPF transmission switching problem is rarely solved due to its high computational intensity. In practice, the OPF is solved using the DC approximation and iterations that try to obtain AC feasibility and N-1 reliability. In order to solve the nonlinear ACOPF with binary variables, the solver could possibly branch on every combination of opened/closed lines in the network and solve a nonlinear program (NLP) for each different instance, which is prohibitively computationally expensive at the current time. If there are N lines, this could require solving 2^N nonlinear programs. In addition, the nonlinear program is nonconvex.

Mixed integer linear programs (MIPs) generally solve much faster than mixed integer nonlinear programs. However, even solving a MIP can also be quite time-intensive, as one may need to solve the linear program (LP) 2^N times; however, techniques for solving MIPs have significantly reduced this upper bound.

One way to limit the number of branches is to constrain the number of lines allowed to be open. Although not necessarily optimal, the approach finds a better solution than the status quo (See Hedman et al, Fuller et al, and Ruiz et al, 2012) and is often very close to the optimal solution. Our work in this paper also shows that opening more lines past a certain number has no or little benefit.

If the system is forced to open S lines, then the number of branches for LPs in the MIP is N choose S. However, in the DC model, it is never optimal to open lines that island parts of the network (see Ostrowski et al, 2012). In a connected network, you can travel from any bus of the network to any other bus via transmission lines (although one may have to travel through other buses). "Islanding" means that there are pairs of buses that are not connected by any path. Aka, a connected network would be one circle, and a network with islanding would be two circles; you could never travel from a bus in the first circle to the bus in the second circle. Therefore, one can reduce the MIP branch and bound tree by removing any paths that result in islanding. To a first order approximation, we believe this is true for AC networks. In this paper, we have not implemented this procedure.

An additional issue is that our linear program is not static; rather, it is based on the last iteration (see O'Neill et al, 2012 and Campaigne et al, 2013). Our linearization depends on the previous optimal point. However, we may then switch the network based on a revised linearization in a later iteration.

This paper seeks to address the following questions:

- Does the linearized ACOPF select lines to switch in accordance with the nonlinear ACOPF?
- How much faster is the iterative linearized ACOPF compared to the nonlinear ACOPF?
- How well do the proposed heuristics perform?

2. Notation.

Variables and parameters are indexed over buses using subscripts n and m, $n, m \in N$. Here we refer to transmission assets as lines. Transmission lines are indexed by $k, k \in K$, with one terminal of line k being n and the other terminal being m. Set $W(n)$ is the set of lines k that connect to bus n. For a complex variable or parameter, the superscript r denotes the real portion and the superscript j denotes the imaginary portion. For example, if $x = a+jb$, $x^r = a$, $x^j = b$ where $j = (-1)^{1/2}$. The index of a major iteration is h.

Decision Variables

p_n real power injected at bus n

q_n reactive power injected at bus n

$v^r{}_n$ real part of the voltage at bus n

$v^j{}_n$ imaginary part of the voltage at bus n

$i^r{}_n$ real part of the current injection at bus n

$i^j{}_n$ imaginary part of the current injectionat bus n

$i^r{}_k$ real part of the current on line k $i^j{}_k$ imaginary part of the current on line k

z_k binary variable that is 1, if the switch at n for line k is open, and 0, if closed

Parameters

$c^p{}_n(p_n)$ quadratic cost of real power at bus n

$c^q{}_n(q_n)$ quadratic cost of reactive power at bus n

$c^{pl}{}_n(p_n)$ stepwise linear approximation of $c^p{}_n(p_n)$

$c^{ql}{}_n(q_n)$ stepwise linear approximation of $c^q{}_n(q_n)$

b_k susceptance of line k

b_{n0} self susceptance of node n (to ground)

g_k conductance of line k g_{n0} self conductance of node n (to ground)

$y_k = g_k + jb_k$ admittance of line k

y_{n0} admittance from bus n to ground

$p_n{}^d$ real power demand at bus n

$q_n{}^d$ reactive power demand at bus n

$p^{min}{}_n$ minimum required real power generation at bus n

$p^{max}{}_n$ maximum allowed real power generation at bus n

$q^{min}{}_n$ minimum required reactive power generation at bus n

$q^{max}{}_n$ maximum allowed reactive power generation at bus n

$v^{min}{}_n$ minimum required voltage magnitude at bus n

$v^{max}{}_n$ maximum allowed voltage magnitude at bus n

$\underline{v}^r{}_n$ real voltage value at bus n from the previous linear program solution

$\underline{v}^j{}_n$ imaginary voltage value at bus n from the previous linear program solution

$\underline{i}^r{}_n$	real current value at bus n from the previous linear program solution
$\underline{i}^j{}_n$	imaginary current value at bus n from the previous linear program solution
$i^{max}{}_k$	maximum current magnitude on line k $\underline{i}^r{}_k$ the real current value on line k from the previous linear program solution
$\underline{i}^r{}_k$	imaginary current value on line k from the previous linear program solution
$\underline{i}^j{}_k$	imaginary current value on line k from the previous linear program solution
M_k	a large constant, used to create an either-or constraint

3. Nonlinear IV-ACOPF Transmission Switching Model

The nonlinear current-voltage (IV)-ACOPF is used as a benchmark for linear approximation ILIV-ACOPF. The IV-ACOPF formulation is

Minimize $\sum_n c^p{}_n(p_n)+c^q{}_n(q_n)$ (1)

Subject to

$i^r{}_k = g_k(v^r{}_n - v^r{}_m) - b_k(v^j{}_n - v^j{}_m)$ for all k (2)

$i^j{}_k = b_k(v^r{}_n - v^r{}_m) + g_k(v^j{}_n - v^j{}_m)$ for all k (3)

$i^r{}_n = \sum_{k\in W(n)} i^r{}_k - g_{n0}v^r{}_n + b_{n0}v^j{}_n$ for all n (4)

$i^j{}_n = \sum_{k\in W(n)} i^j{}_k - g_{n0}v^j{}_n - b_{n0}v^r{}_n$ for all n (5)

$p_n = v^r{}_n i^r{}_n + v^j{}_n i^j{}_n + p_n{}^D$ for all n (6)

$p^{min}{}_n \leq p_n \leq p^{max}{}_n$ for all n (7)

$q_n{}^G = v^j{}_n i^r{}_n - v^r{}_n i^j{}_n + q_n{}^D$ for all n (8)

$q^{min}{}_n \leq q_n \leq q^{max}{}_n$ for all n (9)

$(v^r{}_n)^2 + (v^j{}_n)^2 \leq (v^{max}{}_n)^2$ for all n (10)

$(v^{min}{}_n)^2 \leq (v^r{}_n)^2 + (v^j{}_n)^2$ for all n (11)

$(i^r{}_k)^2 + (i^j{}_k)^2 \leq (i^{max}{}_k)^2$ for all k (12)

The network flow equations, (2)-(5) are linear. The upper bounds on the line current magnitudes (12) and voltage magnitudes at buses (10) are circles with their interiors (convex), and thus can be approximated to any degree of accuracy with circumscribing polygons. The lower bound on voltage magnitude, while non-convex, is seldom binding because the optimization pushes voltages higher to reduce losses. In rectangular form, the equations for real (6) and reactive power (8) injections and withdrawals in terms of current and voltage are second-order non-convex polynomials. The IV-ACOPF with transmission switching is formed by adding binary decision variable z_k and replacing equations (2), (3), and (12) with equations (2.1), (2.2), (3.1), (3.2), and (12.1). z_k=1 if line k is disconnected and z_k=0 if line k is connected.

$i^r{}_k \leq g_k(v^r{}_n - v^r{}_m) - b_k(v^j{}_n - v^j{}_m) + z_k M$ (2.1)

$i^r{}_k \geq g_k(v^r{}_n - v^r{}_m) - b_k(v^j{}_n - v^j{}_m) - z_kM$ (2.2)

$i^j{}_k \geq b_k(v^r{}_n - v^r{}_m) + g_k(v^j{}_n - v^j{}_m) + z_kM$ (3.1)

$i^j{}_k \leq b_k(v^r{}_n - v^r{}_m) + g_k(v^j{}_n - v^j{}_m) - z_kM$ (3.2)

$(i^r{}_k)^2 + (i^j{}_k)^2 \leq (1\text{-}z_k)(i^{max}{}_k)^2$ (12.1)

4. Iterative Linear IV-ACOPF Transmission Switching Model

We approximate the quadratic constraint equations (which express real and reactive power in terms of currents and voltages) with hyperplanes that are tangent to the constraint hypersurfaces using first order Taylor approximations.

With the resulting linear approximations, the ILIV-ACOPF(h) at each major iteration h is:

Minimize	$\sum_n c^{pl}{}_n(p_n)+c^{ql}{}_n(q_n)$		(21)
Subj.	$i^r{}_k \leq g_k(v^r{}_n - v^r{}_m) - b_k(v^j{}_n - v^j{}_m) + z_kM$	for all k	(2.1)
to	$i^r{}_k \geq g_k(v^r{}_n - v^r{}_m) - b_k(v^j{}_n - v^j{}_m) - z_kM$		(2.2)
	$i^j{}_k \geq b_k(v^r{}_n - v^r{}_m) + g_k(v^j{}_n - v^j{}_m) + z_kM$		(3.1)
	$i^j{}_k \leq b_k(v^r{}_n - v^r{}_m) + g_k(v^j{}_n - v^j{}_m) - z_kM$		(3.2)
	$i^r{}_n = \sum_{k\epsilon W(n)} i^r{}_k - g_{n0}v^r{}_n + b_{n0}\, v^j{}_n$	*for all n*	*(4)*
	$i^j{}_n = \sum_{k\epsilon W(n)} i_k - g_{n0}v^j{}_n - b_{n0}\, v^r{}_n$	*for all n*	*(5)*
	$p_n{}^G = \underline{v}^r{}_n i^r{}_n + \underline{v}^j{}_n i^j{}_n + v^r{}_n \underline{i}^r{}_n + v^j{}_n \underline{i}^j{}_n - (\underline{v}^r{}_n \underline{i}^r{}_n + \underline{v}^j{}_n \underline{i}^j{}_n) + p_n{}^D$	for all n	(26)
	$p_n{}^{min} \leq p_n{}^G \leq p_n{}^{max}$	for all n	(27)
	$q_n{}^G = \underline{v}^j{}_n i^r{}_n - \underline{v}^r{}_n i^j{}_n - v^r{}_n \underline{i}^j{}_n + v^j{}_n \underline{i}^r{}_n - (\underline{v}^j{}_n \underline{i}^r{}_n - \underline{v}^r{}_n \underline{i}^j{}_n) + q_n{}^D$	for all n	(28)
	$q_n{}^{min} \leq q_n{}^G \leq q_n{}^{min}$	for all n	(29)
	$\underline{v}^{rf}{}_n v^r{}_n + \underline{v}^{jf}{}_n v^j{}_n \leq (v^{max}{}_n)^2$	for f= 0 1, ..., f^{max} and all n	(30)
	$\underline{v}^{rd}{}_n v^r{}_n + \underline{v}^{jd}{}_n v^j{}_n \leq (v^{max}{}_n)^2$	for d= 0, ..., h-1 and all n	(31)
	$\underline{i}^{rf}{}_k i^r{}_k + \underline{i}^{jf}{}_k i^j{}_k \leq (1\text{-}z_k)\,(i^{max}{}_k)$	for f= 1, ..., f^{max} and all k	(32)
	$\underline{i}^{rd}{}_k i^r{}_k + \underline{i}^{jd}{}_k i^{jb}{}_k \leq (i^{max}{}_k)^2$	for d= 0, ..., h-1 and all k	(33)
	$-2\underline{v}^r{}_n\,(a/h^b) \leq v^r{}_n\text{-}\underline{v}^r{}_n \leq 2\underline{v}^r{}_n(a/h^b)$	for all n	(34)
	$-2\underline{v}^j{}_n\,(a/h^b) \leq v^j{}_n\text{-}\underline{v}^j{}_n \leq 2\underline{v}^j{}_n\,(a/h^b)$	for all n	(35)

where f^{max} is the number of sides of the preprocessed circumscribing polygons and d indexes the iterative tight cuts.

To limit the number of lines to be opened, we add the constraint:

$\sum_k z_k \leq k'$ (36)

The network flow equations, (22)-(25) are linear and unchanged. For the voltage magnitudes, the preprocessed upper bounds are in (30) and the iterative tight cuts are in (31). For the line current magnitudes, the preprocessed upper bounds are in (32) and the iterative tight cuts are in (33).

In rectangular form, real (26) and reactive power (28) injections and withdrawals in terms of current and voltage are approximated by the first order Taylor series.

Here we use the formulation that removes the entire transmission element from the network; no current can flow on the line. If we open only one of the switches, the line becomes a capacitor.

The **iterative linearization method** used to solve the problem is as follows:

1) Set h = 0. Choose a starting point $\underline{v}^{r0}_n$ and $\underline{v}^{i0}_n$. Add a circumscribing polygon for each maximum voltage magnitude and maximum current magnitude constraint. Approximate P and Q with hyperplanes that are tangent to the constraint hypersurfaces.

2) Set h = h+1. Solve the resulting LIV-ACOPF(h) to obtain optimal values for the current and voltage, $\underline{v}^{rh}_n$, $\underline{v}^{jh}_n$, $\underline{i}^{rh}_n$, $\underline{i}^{jh}_n$

3) Check the result for convergence of the optimal values using the actual nonlinear P and Q equations (6) through (12). If within tolerance, stop; otherwise continue.

4) Add another set of tight voltage and current constraints at the optimal voltage solution to further cut off infeasible voltage and current solutions. Relinearize the p and q approximation using the current answer and adjust the stepsize range of the bus voltage. Go to step 2.

The convergence criteria is as follows: if the percent violations of real power (7), reactive power (9), the voltage (10), and the current (12) constraints is under a certain threshold, and the sum of the average percent violations of the voltage, real power, and reactive power is also under a threshold, the solution is determined to be AC feasible. If these violation criteria are not met, the solution is determined not to be AC feasible.

5. Computational Testing

Problems. The test problems consistent of the 14, 30, 57, and 118 bus IEEE test cases (see Table 1) at http://www.ee.washington.edu/research/pstca/index.html. Single-line diagrams of the 14 and 30 bus cases are shown in Figures 1 and 2. The quadratic generator costs come from MATPOWER (Zimmerman et al, 2011). We formulate the 20-step linear approximation to the quadratic function. Where there are multiple transmission lines between two nodes, the lines are aggregated into an equivalent single line between the two nodes. Each test problem has two levels (tight and loose) of line current constraints (see Lipka et al, 2013).

Table 1: IEEE Test Bus System Data

Buses	Lines	Generators		Total Demand	Best Known Value Tight Current Limit		Best Known Value Loose Current Limit	
		No.	Capacity		quadratic	linear	quadratic	linear
14	20	5	7.724	2.590	105.4	107.4	85.3	86.5
30	41	6	326.80	42.42	5.89	6.10	5.79	5.98
57	80	7	326.78	235.26	421.5	432.2	419.2	425.5
118	186	54	99.66	42.42	1364.9	1388.4	1300.1	1315.5

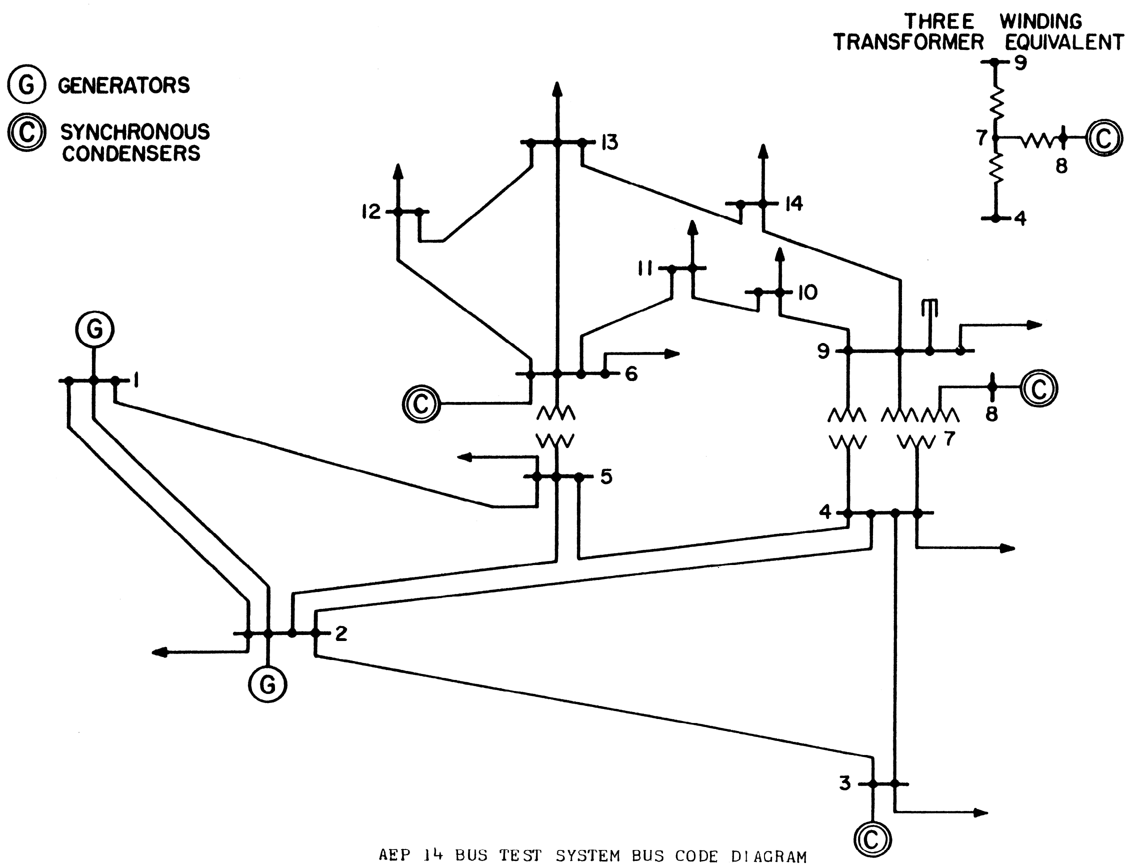


Figure 1: 14 Bus Diagram

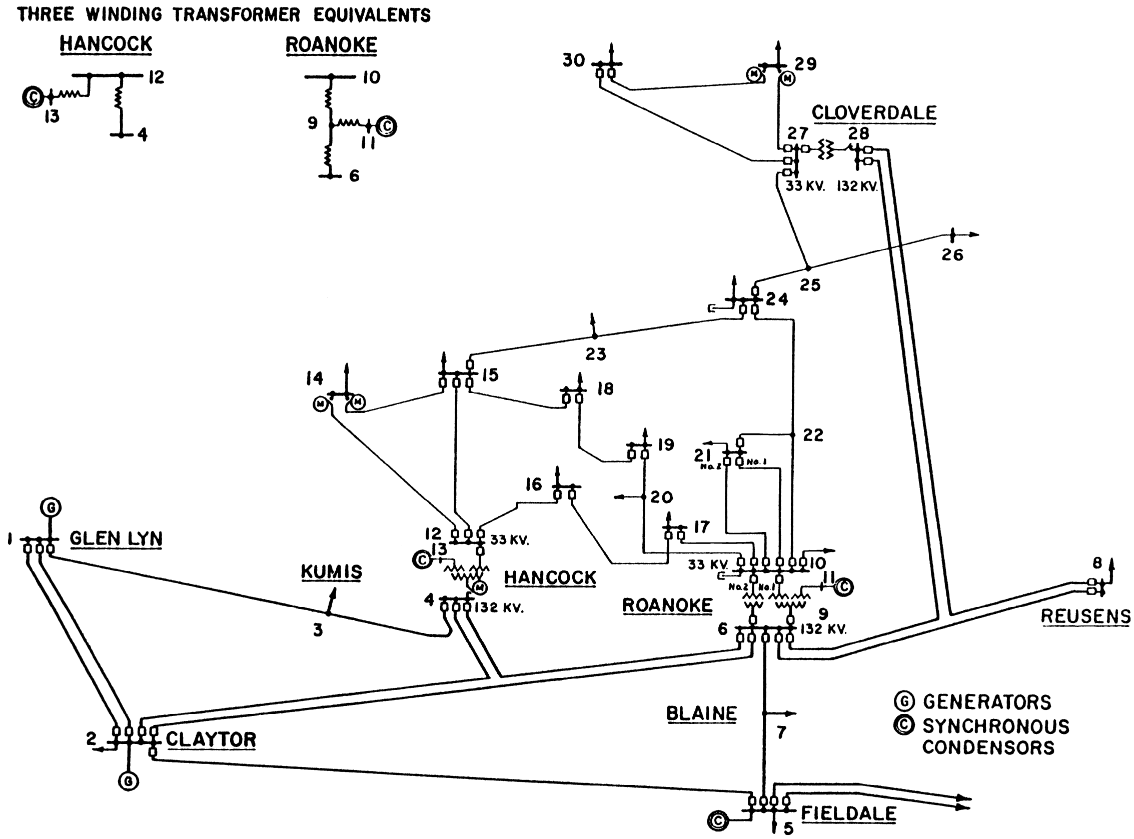


Figure 2: 30 Bus Diagram

Hardware and Software. The problems were solved on an Intel Xeon E7458 server with 8 64-bit 2.4GHz processors and 64 GB memory. However, all problems only used one processor at a time. Minor differences in solution times were recorded when the problems were run at different times of day, but the differences were small enough to be considered background noise. The problems were formulated in GAMS 23.6.2. The nonlinear mixed integer program used was KNITRO. The nonlinear programs without integer variables used solver IPOPT version 3.8. Linear programs used GUROBI version 4.0.0 with the aggressive presolve option. The implementation was simplistic in that the problem was solved from scratch at each major iteration. Starting from the previous linear program was not an option in the GAMS solver. There may be easily gained speedups by not starting each major iteration from scratch. The allowable MIP gap was set to 0.1%.

Optimization Parameters Settings. For IPOPT, we use the default parameters. For step-size constraints, we examine b=1 (linear step size) and b=2 (quadratic step size) and a = 0.5 and 1 in (34) and (35). We choose 16 and 32 sided circumscribing polygons based on the testing in Pirnia et al (2012).

The maximum number of major iterations (linear program) was set to 20. Earlier testing revealed that the solution usually converged to a feasible solution (within tolerance) in 20 iterations or less, and problems that ran for more than 20 iterations would run to the maximum number of allowed iterations. The linear problem is considered feasible within tolerance when the average deviation of voltages, real power, and reactive power is less than 0.1% and the maximum deviation of all individual nodes is less than 0.5%.

Initialization. As the initial starting point, we use a flat start, $\underline{v}^r_n = 1$ and $\underline{v}^i_n = 0$, for all buses n. Other starting points can be used: these include hot starts, random starts, and DC starts (see Castillo et al 2013). In commercial practice, a hot start may be available from previous solutions since each dispatch is solved in sequential time steps.

Termination. The maximum number of major iterations was set to 100. The relative convergence tolerances were set at 0.001 or 0.005. The maximum CPU time was never reached.

We examine five different approaches: brute force (total enumeration) one-line optimization, MIP one-line optimization, iterative linearized ACOPF with a MIP at every iteration (opening up to 5 lines), linearized ACOPF with the MIP at the first iteration (opening up to 5 lines), and a progressive approach where one additional line is allowed to be open at each stage (up to 5 lines).

Brute-Force One-Line Approach. First, we examine the results of the linearized ACOPF versus the nonlinear ACOPF when the network is fixed to one line removed using the following procedure, where N subproblems are solved.

The procedure in steps 1-5 is repeated for 4 different approaches: 16 and 32 preprocessed voltage cuts with linear and quadratic voltage step size reduction.

1) For each line k in the transmission system with K lines:
2) In the model described in section 4, z is a parameter instead of a variable. Let all lines be in service (aka, z=0) except for line k (set z=1). Note that since z is a parameter instead of a variable, an LP instead of a MIP is solved.
3) Use the iterative linearization method to solve the problem.
4) Set line k to be line $k+1$. Go to step 2 if k is less than or equal to K.
5) When all of the results have been tabulated for each line taken out ($k=K+1$), the subproblem with the lowest cost is considered to be the optimal configuration.
6) To obtain a nonlinear comparison, steps 1-5 are repeated except in step 2, the nonlinear model described in section 3 is used and step 3 is replaced with the solver's nonlinear solution methods.

MIP One-Line Approach. We also compare the results of the linearized ACOPF with a MIP at every iteration to the results of brute force linearized ACOPF in order to demonstrate whether the MIP at each iteration works appropriately.

The procedure in steps 1-2 is repeated for 4 different approaches: 16 and 32 preprocessed voltage cuts with linear ($b = 1$) and quadratic ($b = 2$) voltage step size reduction.

1) Use the model described in section 4 with the added constraint $\sum_k z_k = 1$. This means that in any answer, one transmission line will be open.
2) Use the iterative linearization method to solve the problem. At every iteration, a MIP will be solved.
3) Then, to obtain a nonlinear comparison, steps 1 and 2 are repeated except in step 1, the nonlinear model (KNITRO) described in section 3 is used and step 2 is replaced with the solver's nonlinear solution methods.

Iterative Linearized ACOPF with a MIP at every iteration opening up to 5 lines More extensively, the ACOPF was run for opening up to 5 lines. This limit on the number of lines opened was placed to achieve a reasonable run time. Additionally, allowing the program open up to ten lines did not result in significant reduction of the total power production cost versus allowing it to open 5 lines.

The procedure is repeated for 4 different approaches: 16 and 32 preprocessed voltage cuts with linear and quadratic voltage step size reduction.

1) Use the model described in section 4 with the added constraint $\sum_k z_k \leq k'$ *where* $k' = 5$. This means that in any answer, up to 5 transmission lines can be open.
2) Use the iterative linearization method to solve the problem. At every iteration, a MIP will be solved.
3) When the iterative linearization method has either converged or exceeded the iteration limit, solve the nonlinear ACOPF with transmission switching as in section 3 but again the line configuration (z_k) variables are set as parameters (according to the solution in step 2), again to evaluate whether the LIV-ACOPF solution was truly AC feasible.

Linearized ACOPF with the MIP at the first iteration opening up to 5 lines.
To reduce the computational time of solving the ACOPF, a second approach is to run a MIP only on the first iteration of the iterative linearization method, then run the other LP iterations on the fixed network that was the optimal answer from the MIP repeatedly. The steps used are as follows:

The procedure is repeated for 4 different approaches: 16 and 32 preprocessed voltage cuts with linear and quadratic voltage step size reduction.

1) Use the model described in section 4 with the added constraint $\sum_k z_k \leq k'$ *where* $k' = 5$. This means that in any answer, up to 5 transmission lines can be open.

2) Do one iteration of the iterative linearization method to solve the problem. This iteration will solve a MIP.

3) In the model described in section 4, replace the variables z with parameters. The line configuration will be set to the optimal solution found in step 2. For example, if step 2 found that all lines are in the network except the lines from bus 1 to 4 and from bus 2 to 5, then $z_{141} = 1$, $z_{251} = 1$, and all other $z = 0$.

4) Solve the model in step 3 using the iterative linearization method.

5) When the iterative linearization method has either converged or exceeded the iteration limit, solve the nonlinear ACOPF with transmission switching as in section 3, but again the line configuration (z_k) variables are set as parameters (according to the solution in step 2), again to evaluate whether the LIV-ACOPF solution was truly AC feasible.

It is expected that this model will take less time than the previous model, since less MIPs will be run during the process.

Progressive MIP. The purpose of this method is to reduce the number of branches the MIP has to consider in the case of opening up to 5 transmission lines. The previous approach has (N choose 5)+(N choose 4)+(N choose 3)+(N choose 2)+(N choose 1) potential branches. In this approach, at each iteration k, with the first iteration as k=0, the algorithm decides which of N-k lines to open or whether to not open any additional lines, with a total of N-k+1 possibilities. This approach has up to $(N+1)+(N)+(N-1)+(N-2)+(N-3)=5N-5$ potential branches, which is many fewer than the previous approach. Table 2 shows this significant reduction.

Table 2. Number of Potential MIP branches: Normal vs. Progressive

		Number of Potential Branches	
Buses	Lines	Normal MIP	Progressive MIP
14	20	21,700	95
30	41	862,190	200
57	86	37,055,939	425
118	186	1,806,641,034	925

The following procedure is repeated for 4 different approaches: 16 and 32 preprocessed voltage cuts with linear and quadratic voltage step size reduction.

1) Use the model described in section 4 with the added constraint $\sum_k z_k \leq k'$ where $k' = 1$. This means that in any answer, only 1 transmission line can be open.

2) Use the iterative linearization method to solve the problem. This iteration will solve a MIP with N+1branches.

3) If a line is opened, fix it as a parameter (aka, z=1). Now, let k'=2. Note that since one line is open, only one more line can possibly be opened.
4) Again, use the iterative linearization method to solve the problem.
5) Fix all open lines as parameters, and increment k'=k'+1.
6) Repeat steps 4 and 5 until the problem where k'=5 has been solved.
7) When the iterative linearization method has either converged or exceeded the iteration limit, solve the nonlinear ACOPF with transmission switching as in section 3, but again the line configuration (z_k) variables are set as parameters (according to the solution in step 2). This is done to evaluate whether the LIV-ACOPF solution was truly AC feasible.

5.1 Brute-Force versus MIP One Line Approach Results

To benchmark the linearized ACOPF program, both the linearized and nonlinear ACOPF programs were solved on a fixed configuration of open/closed lines for all possible configurations of one line open and all others closed (the *Brute Force* Method). The objective function values of all these configurations are compared, and the 'best' answer is the problem with the lowest objective function value. This *Brute Force* approach is compared to solving the problem with the linearized ACOPF and allowing the mixed integer solver to find the best network configuration with one line out (MIP). The nonlinear "brute force" method uses IPOPT to solve every possible network configuration with one line out, and the objective value given is the minimum cost of all of these configurations. The nonlinear "MIP" method uses KNITRO to solve the nonlinear MIP with one open line.

14-bus problem. The results for the 14 bus problem are shown in Tables 3 and 4. The objective function values were within 2% of the best-known value (the nonlinear solution) for both the MIP and Brute Force methods in both the tight and loose current constrained cases. In addition, all of the linear approaches were at least 4 times as fast as the nonlinear case. In the tight current constrained case, the same line (4-7) was switched out by all but one of the 8 approaches. The linear case objective values are all very close to each other – the biggest difference is 0.1%. Removing line 4-7 was highly beneficial in the tightly constrained case; the total generation cost was reduced by nearly 10%. In the loose current constrained case, again the same line (2-5) was selected to be switched by all but one of the 8 approaches. However, the objective values of the linear approaches differ more than in the tight current case; the biggest difference is 3.2%. While the nonlinear case shows a small improvement in the total generation cost by switching out a line, some of the linear approaches show an improvement while some do not. In the tightly constrained case, the MIP runs faster than the brute force method. Surprisingly, in the loosely constrained case, the MIP runs slower than the brute force method; this is likely not significant due to run-to-run timing variability.

Table 3. One Line Results: 14 Bus, Tight Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	105.69	106.53	105.69	106.53	107.36
MIP Opening One Line	Obj Value	95.05	95.04	95.05	95.04	93.77
	Line Switched	4-7	4-7	4-7	4-7	4-7
	CPU Time	6.25	6.38	5.18	9.32	54.30
Brute Force Opening One Line	Obj Value	95.04	94.99	95.00	94.97	94.09
	Line Switched	4-7	2-5	4-7	4-7	4-7
	CPU Time	6.81	9.44	10.85	14.59	192.12

Table 4. One Line Results: 14 Bus, Loose Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	85.94	85.63	85.15	86.13	86.51
MIP Opening One Line	Obj Value	82.80	85.26	85.54	85.55	84.07
	Line Switched	2-5	2-3	2-5	2-5	2-5
	CPU Time	10.89	22.06	5.80	9.80	100.78
Brute Force Opening One Line	Obj Value	84.94	84.85	85.28	85.29	84.42
	Line Switched	2-5	2-5	2-5	2-5	2-5
	CPU Time	8.94	9.47	5.11	6.97	63.52

30 bus problem. The results for the 30 bus problem are shown in Tables 5 and 6. In both the tightly and loosely constrained cases, the biggest difference between the objective values in the linear approaches is 2%. In both tight and loose current constraint cases, it appears that removing one line does improve the objective value, although this improvement is very small. The MIP and Brute Force methods choose different lines to open in the same problem, although each method chooses the same line within the linear approaches. However, choosing the different line does not seem to affect the value of the objective function. Here, the MIP has a clear advantage over the Brute Force method in the solution time. The MIP runs more than twice as fast as the Brute Force method in all approaches except for the loosely constrained case with quadratic step size and 32 cuts. In the brute force method, linear approaches solved at least 4 times as fast as the nonlinear approach. Since there were different results on different simulations for the MIP with the nonlinear case, the simulation result of the last run is captured here.

Table 5. One Line Results: 30 Bus, Tight Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	6.02	6.08	6.02	6.08	6.10
MIP Opening One Line	Obj Value	5.82	5.91	5.87	5.91	5.74
	Line Switched	25-27	25-27	25-27	25-27	25-27
	CPU Time	37.05	40.49	20.71	34.85	536.4
Brute Force Opening One Line	Obj Value	5.93	5.93	5.92	5.93	5.79
	Line Switched	6-28	6-28	6-28	6-28	6-28
	CPU Time	124.8	117.2	93.19	113.6	554.5

Table 6. One Line Results: 30 Bus, Loose Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	5.96	5.98	5.96	5.98	6.00
MIP Opening One Line	Obj Value	5.81	5.81	5.81	5.89	5.74
	Line Switched	25-27	25-27	25-27	25-27	24-25
	CPU Time	30.81	66.29	21.67	29.90	633.7
Brute Force Opening One Line	Obj Value	5.92	5.92	5.92	5.92	5.78
	Line Switched	6-28	6-28	6-28	6-28	24-25
	CPU Time	75.39	156.0	89.76	53.73	683.1

57 bus problem. The results for the 57-bus problem are shown in Tables 7 and 8. 8The lines chosen to open are not very consistent between the different approaches, suggesting that the objective function has several topologies with similar objective values. The objective function values were within 0.2% of the best-known value with the brute force method. Here, the advantage of the MIP over the Brute Force solution time increases; the MIP solves more than 25 times faster than the brute force method. However, the Brute Force method appears to find better solutions; all solutions of the brute force method are an improvement over not switching any lines, and all MIP solutions have greater generator cost than the Brute Force solutions.

Table 7. One Line Results: 57 Bus, Tight Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	424.02	422.71	424.07	423.69	433.85
MIP	Obj Value	424.66	426.98	426.99	426.75	419.08
Opening One Line	Line Switched	56-57	38-48	2-3	2-3	9-13
	CPU Time	34.82	54.85	12.15	21.10	3000.1
Brute Force	Obj Value	421.16	421.65	421.49	421.70	420.71
Opening One Line	Line Switched	12-16	38-49	9-11	9-13	48-49
	CPU Time	1,298.15	1,708.27	815.8	1,034.99	10,966.5

Table 8. One Line Results: 57 Bus, Loose Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	422.60	422.42	423.27	423.47	425.48
MIP	Obj Value	426.33	426.98	429.01	426.75	418.47
Opening One Line	Line Switched	38-48	38-48	1-2	2-3	3-4
	CPU Time	34.09	54.85	9.62	21.10	2982.6
Brute Force	Obj Value	421.16	421.65	421.49	421.70	420.71
Opening One Line	Line Switched	12-16	38-49	9-11	9-13	48-49
	CPU Time	1,292.2	1,668.6	768.5	964.6	9,287.3

118 bus problem. The results for the 118-bus problem are shown in Tables 9 and 10. In the tight current constraint case, the linear approaches choose either line 77-80 or line 77-82 to be removed. In the loose current case, there is more deviation in which line is taken out. The MIP is again much faster than the brute force method (in both tightly and loosely constrained cases). Objective function values are within 3.2% of the nonlinear values in the tight case and within 1.5% of the loose case. In the tight current constraint case, the objective values found by the MIP and the brute force method are very similar. In the loose current constraint, the brute force method appears to find better solutions; all the objective values in the brute force method are an advantage over not switching any lines; most of the solutions found by the MIP are more costly than not switching any lines.

Table 9: One Line Results: 118 Bus, Tight Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	1,381.15	1,380.4	1,388.3	1,388.3	1,364.9
MIP	Obj Value	1,379.12	1,378.2	1,385.7	1,385.8	1,344.3
Opening One Line	Line Switched	77-82	77-82	77-82	77-82	17-31
	CPU Time	213.1	307.8	256.6	330.1	30,000
Brute Force	Obj Value	1,379.2	1,378.4	1,384.6	1,384.1	1,368.9
Opening One Line	Line Switched	77-80	77-82	77-82	77-80	77-82
	CPU Time	12,303	14,631	6,534.6	16,175	26,359

Table 10: One Line Results: 118 Bus, Loose Current Constraint

Step Size Function		Linear		Quadratic		
Number of Cuts		16	32	16	32	Nonlinear
Without Opening Lines	Obj Value	1,307.7	1,311.8	1,310.7	1,314.6	1,300.1
MIP	Obj Value	1,314.1	1,315.2	1,314.4	1,314.4	1,296.8
Opening One Line	Line Switched	25-26	25-26	25-26	25-26	24-70
	CPU Time	123.1	186.0	34.56	61.64	322.6
Brute Force	Obj Value	1,305.3	1,304.9	1,305.9	1,307.6	1,304.6
Opening One Line	Line Switched	46-48	100-104	45-46	40-41	19-20
	CPU Time	22,780	34,747	11,874	16,091	21,236

Comparison: One MIP, Repeated MIP, and Progressive MIP

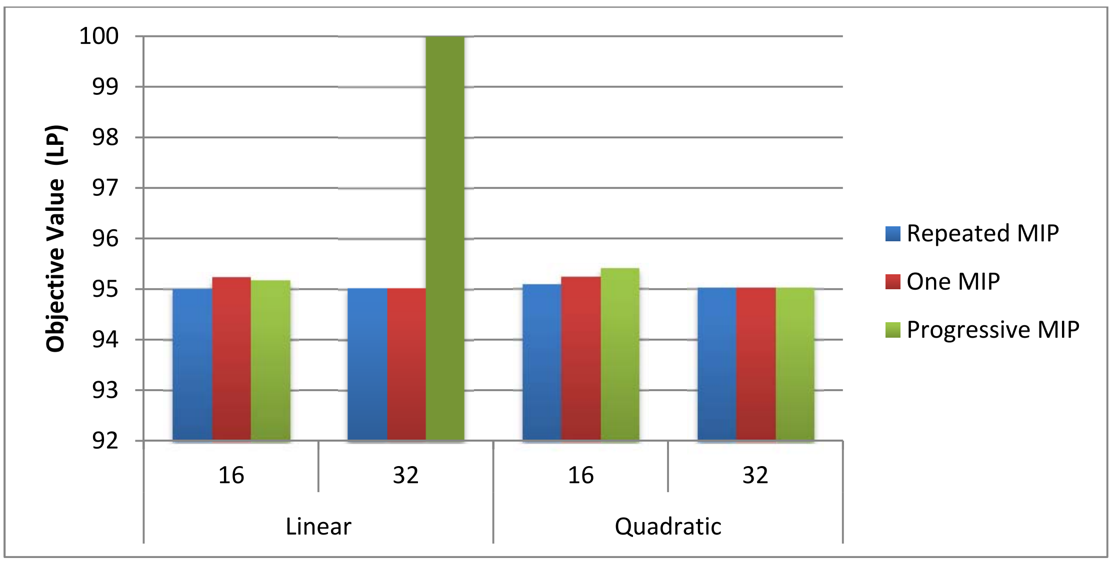
14 Bus, Tightly Constrained

Objective Value Function (LP)

All three methods of solving the MIP reported very similar objective function values, except for the progressive MIP with a linear step size function and 32 cuts, where the answer was infeasible (see Table 11).

Table 11. 14 Bus, Tightly Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	95.02	95.04	95.10	95.04
One MIP	95.25	95.04	95.25	95.04
Progressive MIP	95.19	INF	95.42	95.04
No lines open	105.69	106.53	105.69	106.53



LP vs. NLP Objective Function Value Differences

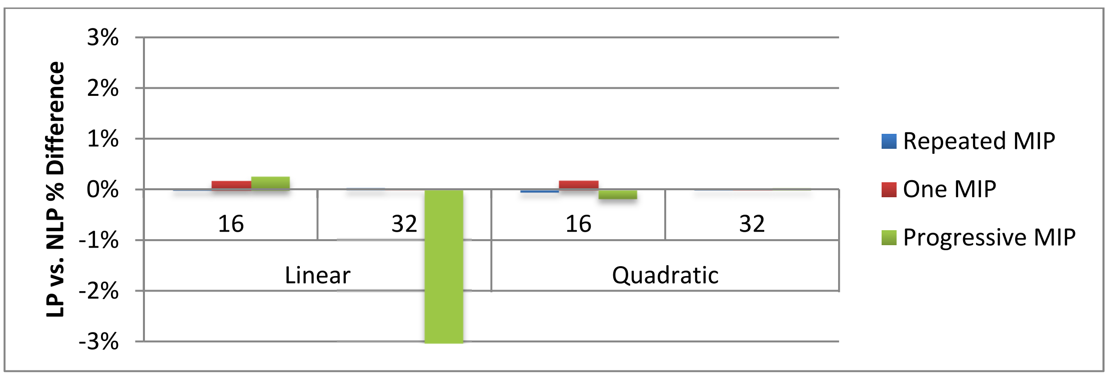
The LP vs. NLP Objective Value Difference is computed as the objective values of the (NLP-LP)/LP. As a reminder, the NLP Objective Value is the answer to the problem where the transmission lines found to be taken out in the LP are fixed using the solver IPOPT, and the ACOPF is solved with the original, nonlinear constraints (but with linear objective function). In this case, as shown in tables 12 and 13, all the nonlinear answers are close to the linear objective value, except for the progressive MIP with 32 cuts and a linear step size. This large discrepancy occurs because the linear method finds an infeasible solution and the nonlinear method finds a feasible solution.

Table 12. 14 Bus, Tightly Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	95.06	95.07	95.06	95.06
One MIP	95.41	95.06	95.42	95.06
Progressive MIP	95.43	94.95	95.26	95.06

Table 13. 14 Bus, Tightly Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	0.04	0.04	-0.05	0.02
One MIP	0.17	0.02	0.18	0.02
Progressive MIP	0.26	INF	-0.17	0.02

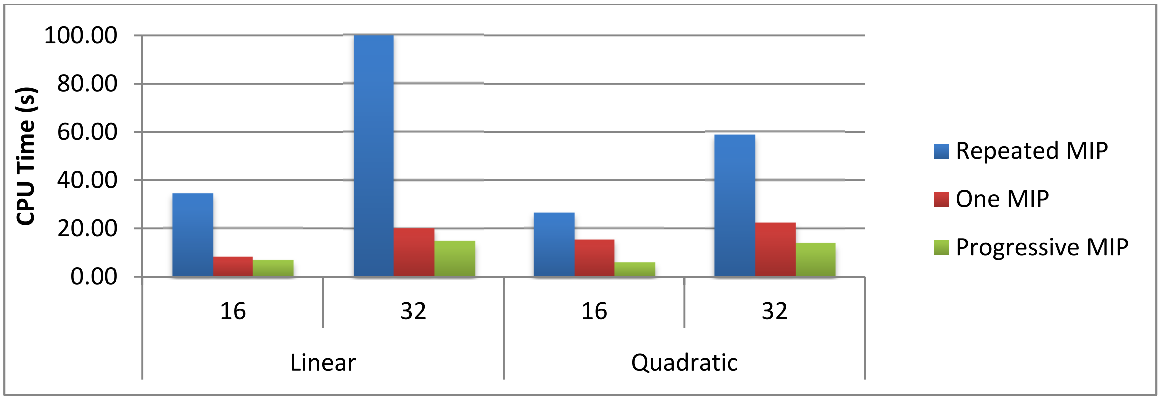


CPU Time

Table 14 shows that using only one MIP at the beginning dramatically reduces the time to solve the switching problem. The progressive MIP has a slight advantage over using one MIP.

Table 14. 14 Bus, Tightly Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	34.68	101.03	26.75	59.09
One MIP	8.36	20.60	15.83	22.56
Progressive MIP	7.11	15.18	6.24	14.15



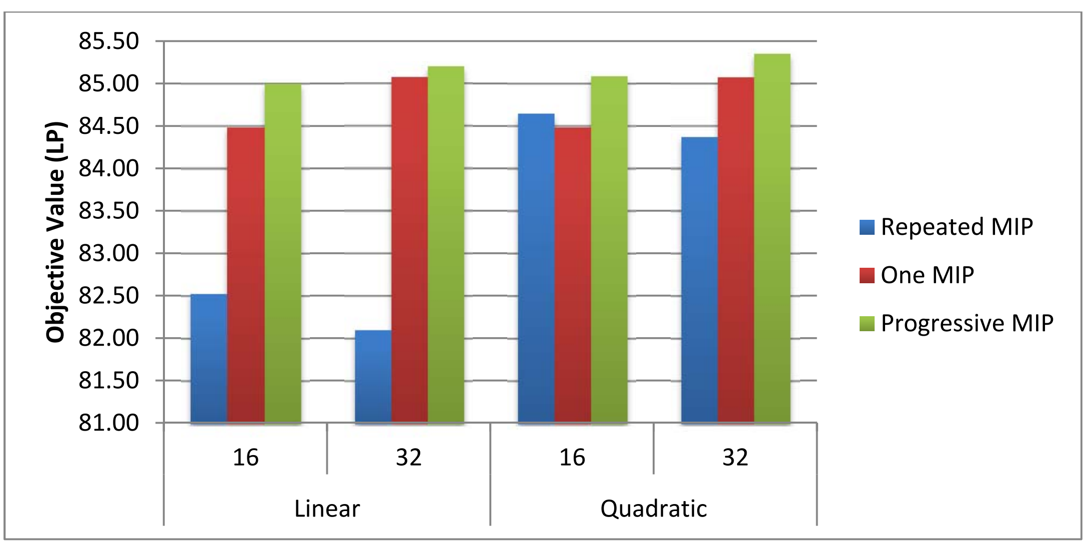
14 Bus, Loosely Constrained

Objective Value Function (LP)

The linear objective values found by all methods are relatively close, as seen in Table 15, except the repeated MIP with linear step size function appears to deviate from all the other methods, although it still finds a feasible solution.

Table 15. 14 Bus, Loosely Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	82.53	82.10	84.65	84.38
One MIP	84.49	85.08	84.49	85.08
Progressive MIP	85.01	85.21	85.09	85.36
No lines open	85.94	85.63	85.15	86.13



LP vs. NLP Objective Function Value Differences

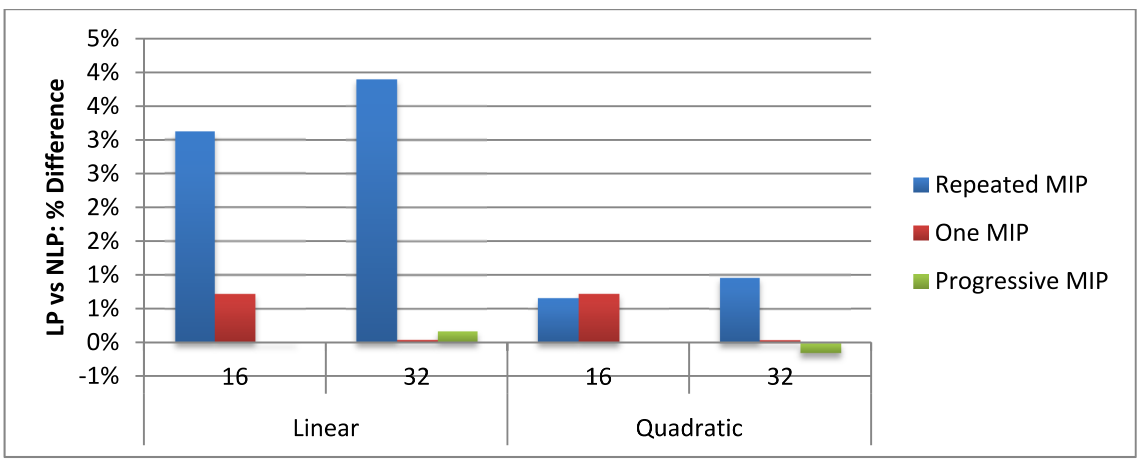
Tables 16 and 17 display that the nonlinear and linear objective values are within 1% except for the repeated MIP method with linear step size reductions. However, the nonlinear objective value for this case is close to 85 rather than 82, which is what the one MIP and progressive MIP methods found.

Table 16. 14 Bus, Loosely Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	85.20	85.43	85.21	85.20
One MIP	85.11	85.12	85.11	85.11
Progressive MIP	85.02	85.36	85.10	85.23

Table 17. 14 Bus, Loosely Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	3.13	3.90	0.66	0.97
One MIP	0.73	0.05	0.73	0.04
Progressive MIP	0.02	0.18	0.01	-0.15

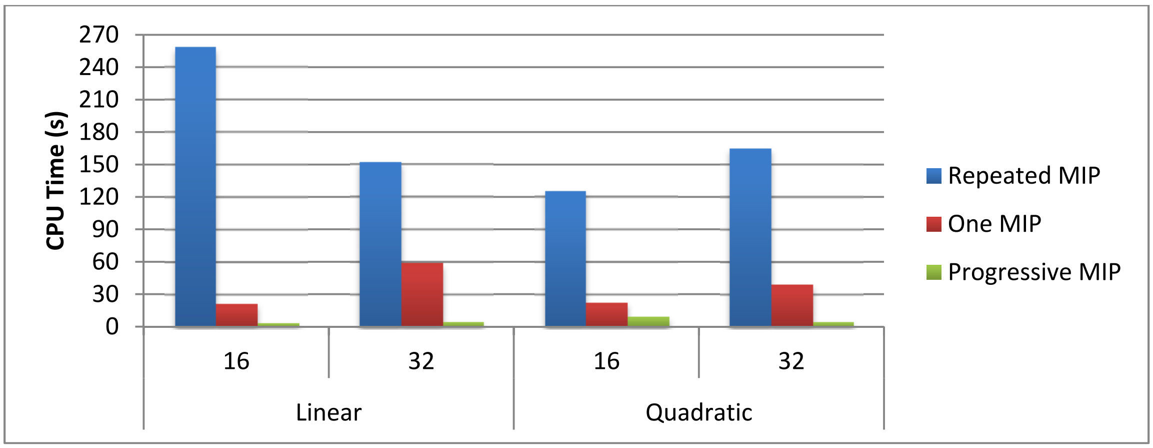


CPU Time

Table 18 shows that the one MIP method solved significantly faster than the repeated MIP method, and the progressive MIP method solved significantly faster than the progressive MIP method. However, there is not a clear pattern which number of cuts or step size function is faster.

Table 18. 14 Bus, Loosely Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	258.65	152.38	126.00	165.26
One MIP	22.08	59.40	23.12	40.16
Progressive MIP	4.21	5.15	9.77	5.14



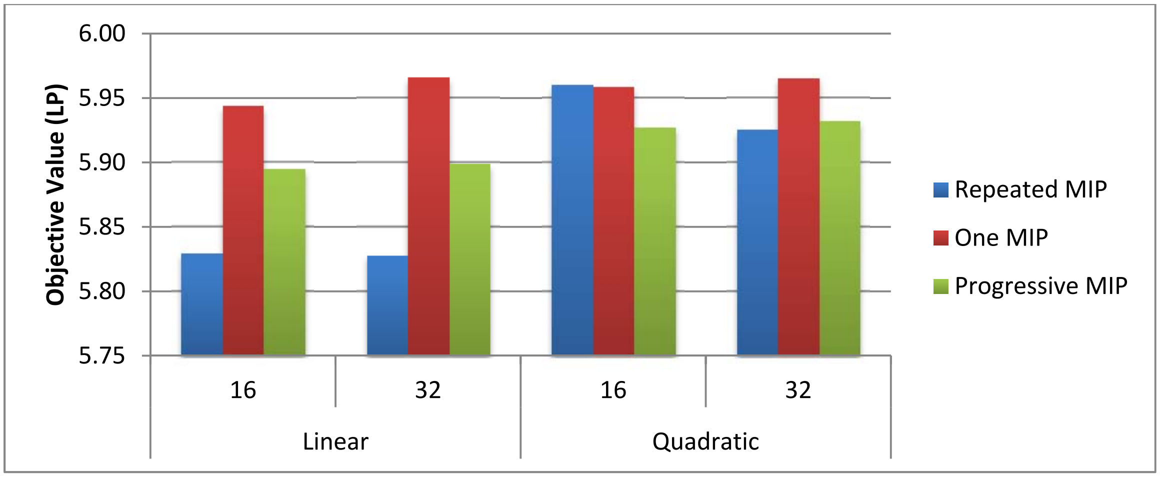
30 Bus, Tightly Constrained

Objective Value Function (LP)

The results of solving the 30 bus system with tight current constraints are given in Table 19. All methods found a feasible answer for this problem. Like the previous problem, the repeated MIP method with linear step size function seemed to find answers that deviated from all the other methods.

Table 19. 30 Bus, Tightly Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	5.83	5.83	5.96	5.93
One MIP	5.94	5.97	5.96	5.97
Progressive MIP	5.89	5.90	5.93	5.93
No lines open	6.02	6.08	6.02	6.08



LP vs. NLP Objective Function Value Differences

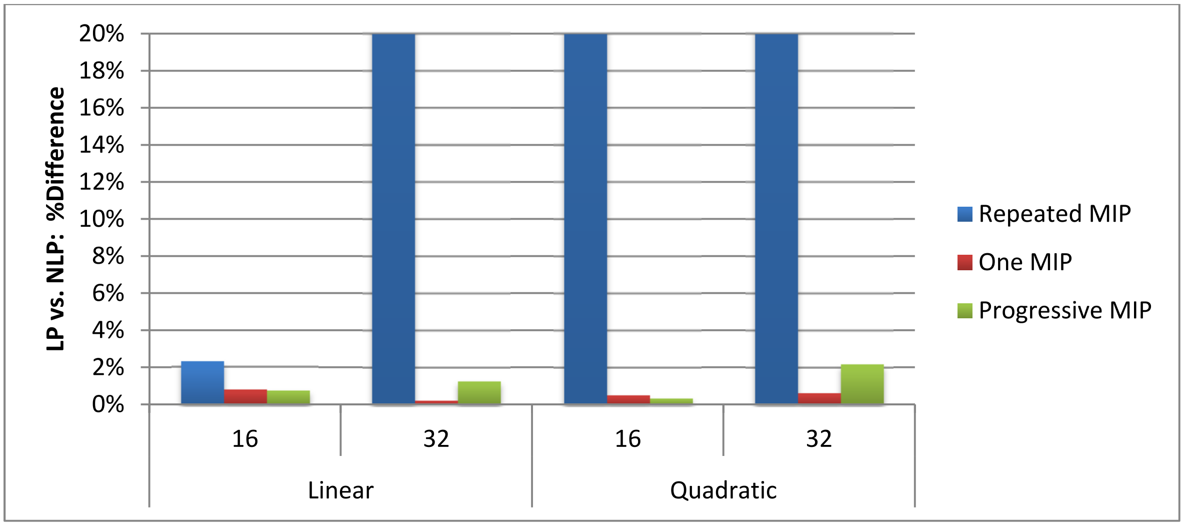
Tables 20 and 21 reveal that the NLP solution was very close to the LP solution in the One MIP and Progressive MIP approaches; however, the network configuration found by the LP in the repeated MIP cases (except for the linear step size with 16 cuts) was infeasible once all the nonlinearities were considered.

Table 20. 30 Bus, Tightly Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	5.97	INF	INF	INF
One MIP	5.99	5.98	5.99	6.01
Progressive MIP	5.94	5.98	5.95	6.07

Table 21. 30 Bus, Tightly Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	2.30	INF	INF	INF
One MIP	0.81	0.25	0.54	0.67
Progressive MIP	0.80	1.31	0.40	2.20

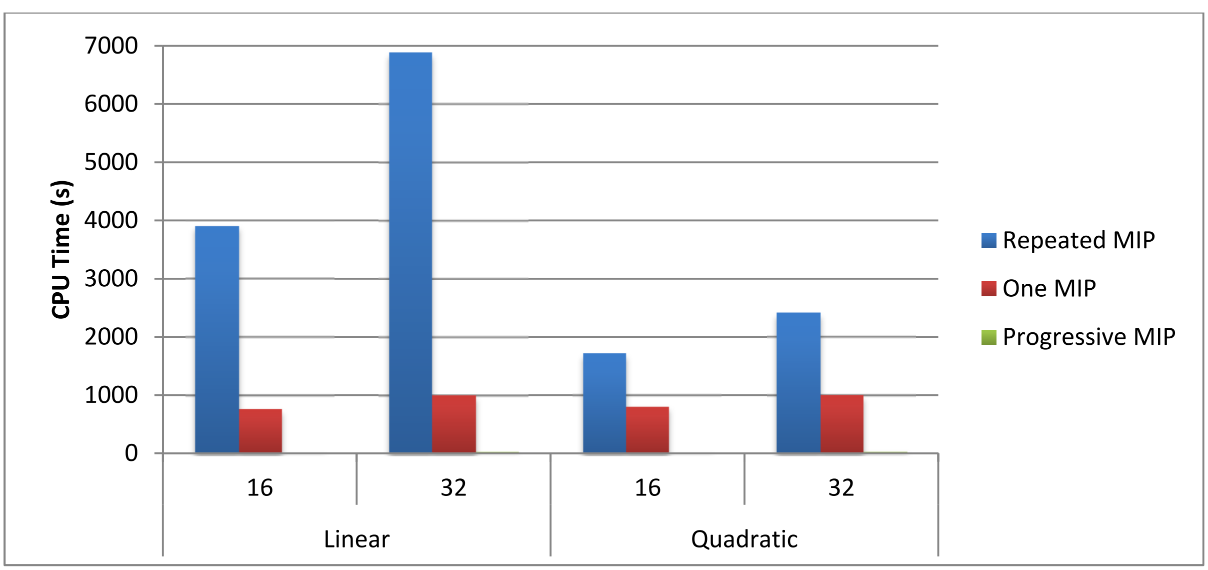


CPU Time

From Table 22, we see that the progressive MIP approach solved significantly faster than the one MIP approach that solved the problem much faster than the repeated MIP approach.

Table 22. 30 Bus, Tightly Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	3908.01	6886.65	1718.02	2409.33
One MIP	763.19	1001.37	806.55	1000.99
Progressive MIP	24.85	34.15	20.71	41.44



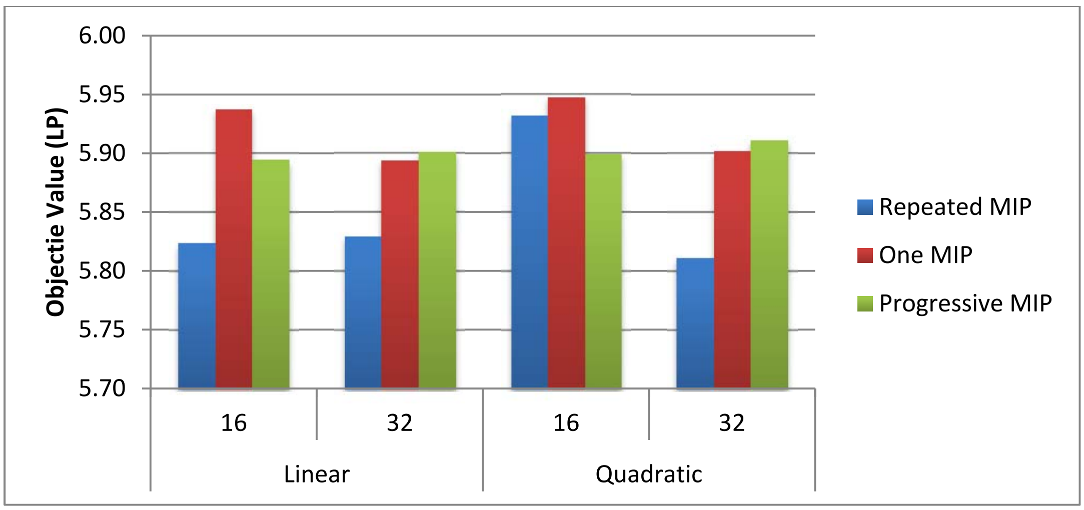
30 Bus, Loosely Constrained

Objective Value Function (LP)

Table 23 shows that the repeated MIP approach (except for the 16 cuts, quadratic step size case) seemed to find significantly different answers than those found by using One MIP and the Progressive MIP.

Table 23. 30 Bus, Loosely Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	5.82	5.83	5.93	5.81
One MIP	5.94	5.89	5.95	5.90
Progressive MIP	5.89	5.90	5.90	5.91



LP vs. NLP Objective Function Value Differences

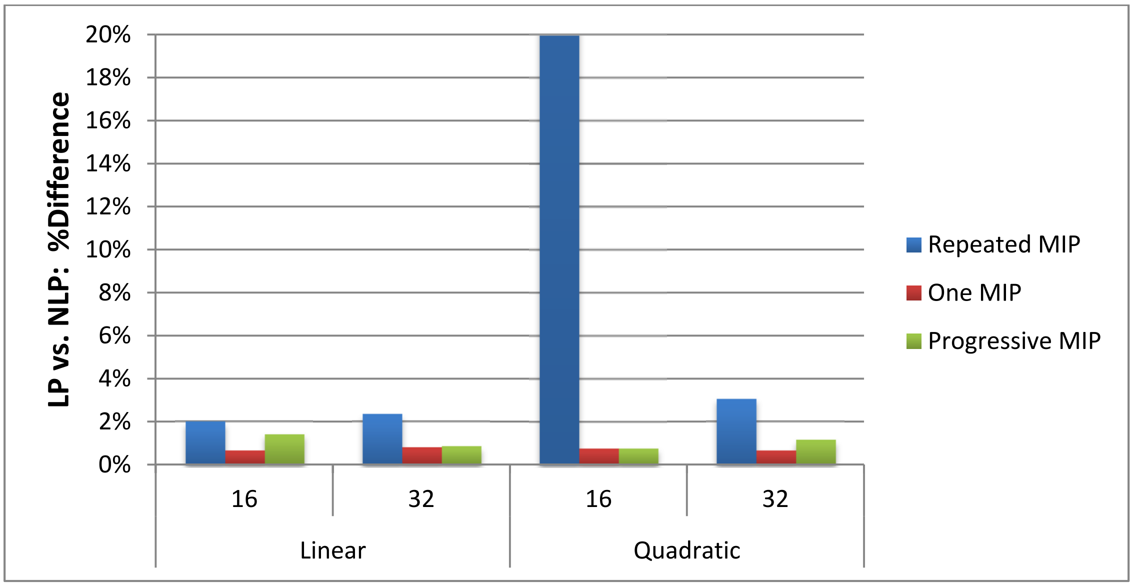
There is only one case where the network configuration found by the linear approach was not feasible – the repeated MIP, quadratic step size, 16 cuts case as seen in Tables 24 and 25. Otherwise, the difference between the answers from the NLP and the LP are 3.1% or less.

Table 24. 30 Bus, Loosely Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	5.94	5.97	INF	6.00
One MIP	5.98	5.94	5.99	5.94
Progressive MIP	5.98	5.95	5.95	5.98
No lines open	5.96	5.98	5.96	5.98

Table 25. 30 Bus, Loosely Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	2.01	2.36	INF	3.09
One MIP	0.67	0.82	0.77	0.70
Progressive MIP	1.43	0.87	0.80	1.16

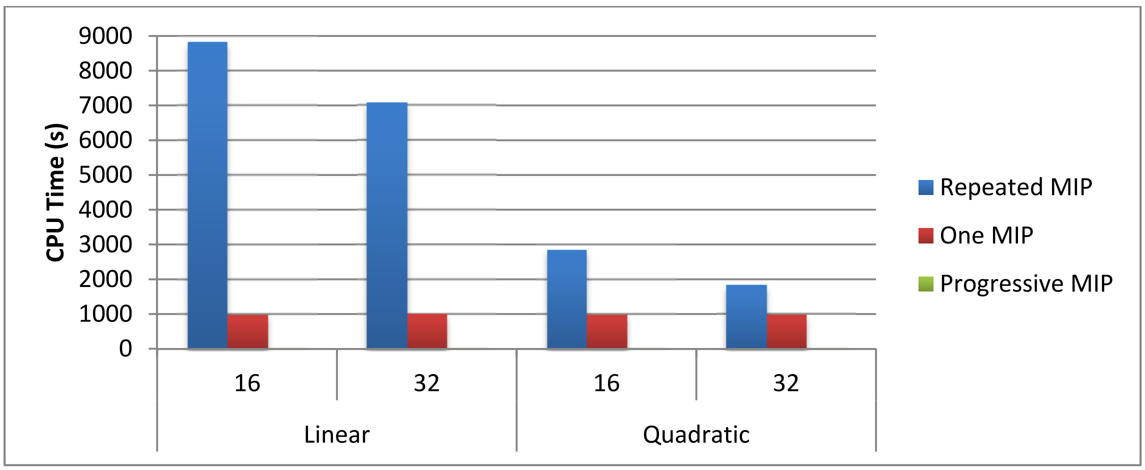


CPU Time

Again, as seen in Table 26, the progressive MIP was much faster than one MIP that was much faster than the progressive MIP. The repeated MIP seemed to run much faster when a quadratic step size was used.

Table 26. 30 Bus, Loosely Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	8834.50	7083.96	2871.75	1831.51
One MIP	1000.54	1002.15	1000.51	1000.98
Progressive MIP	25.99	29.75	20.17	28.86



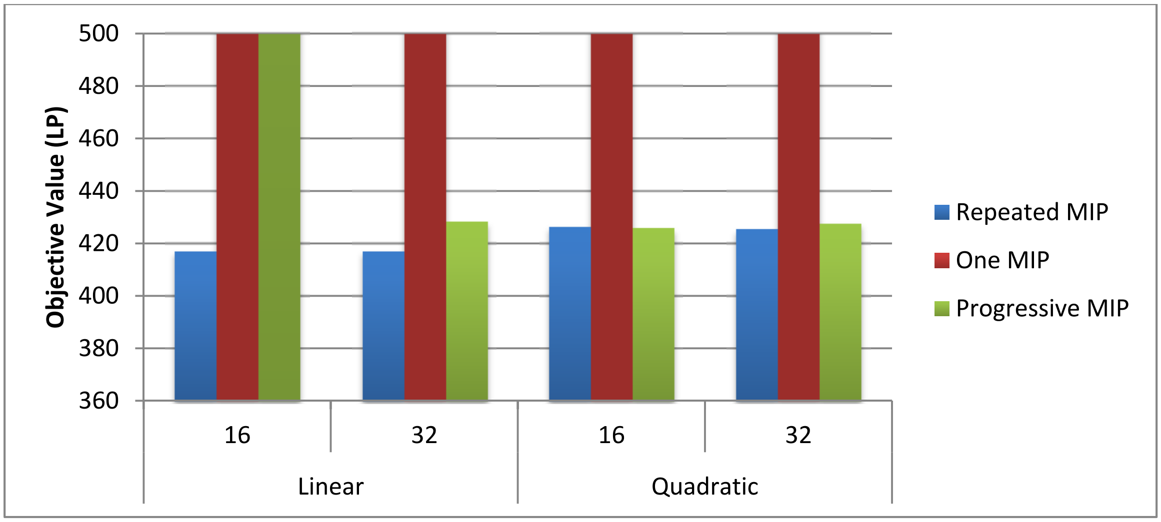
57 Bus, Tightly Constrained

Objective Value Function (LP)

In this case, Table 27 shows that 5 out of the 12 approaches found infeasible solutions. Only the repeated MIP consistently found a feasible solution. The optimal objective value found by the repeated MIP differed greatly between the linear and quadratic case.

Table 27. 57 Bus, Tightly Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	417.41	417.39	426.67	425.75
One MIP	INF	INF	INF	INF
Progressive MIP	INF	428.56	426.15	427.96
No lines open	424.02	422.71	424.1	423.7



LP vs. NLP Objective Function Value Differences

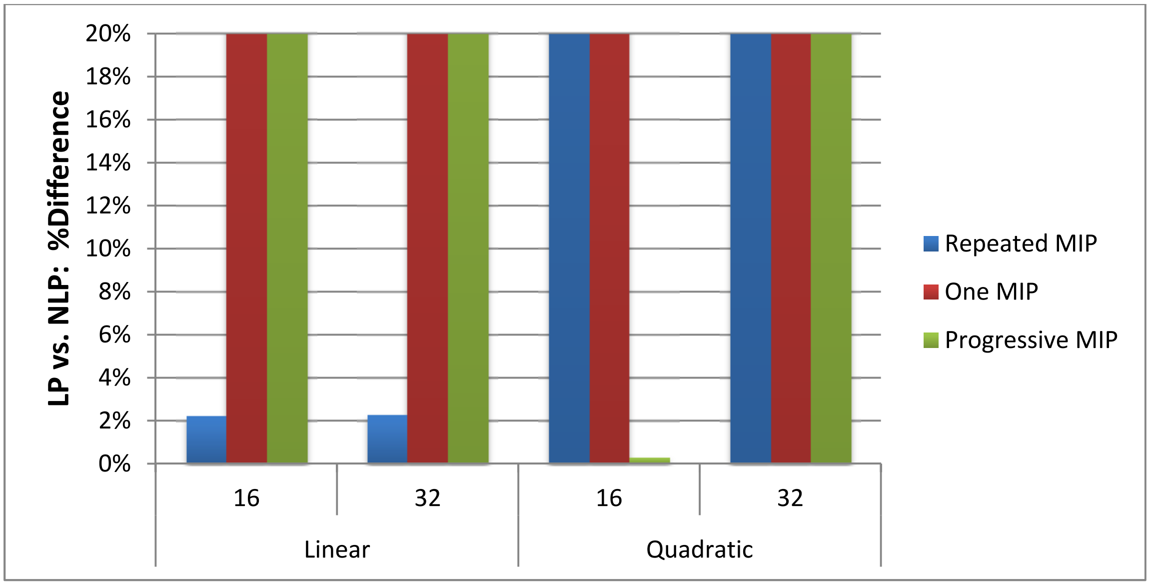
The configurations found by the LP solution were infeasible in 9 out of the 12 different approaches as displayed in Tables 28 and 29. This suggests that the 57 bus problem with a tight current constraint is likely to become infeasible during switching. A constraint enforcing the voltage minimum would need to be added to avoid these infeasibilities.

Table 28. 57 Bus, Tightly Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	426.93	427.25	INF	INF
One MIP	INF	INF	INF	INF
Progressive MIP	INF	INF	427.62	INF

Table 29. 57 Bus, Tightly Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	2.23	2.31	99.98	99.99
One MIP	99.93	99.68	95.29	99.77
Progressive MIP	99.95	85.03	0.34	100.00

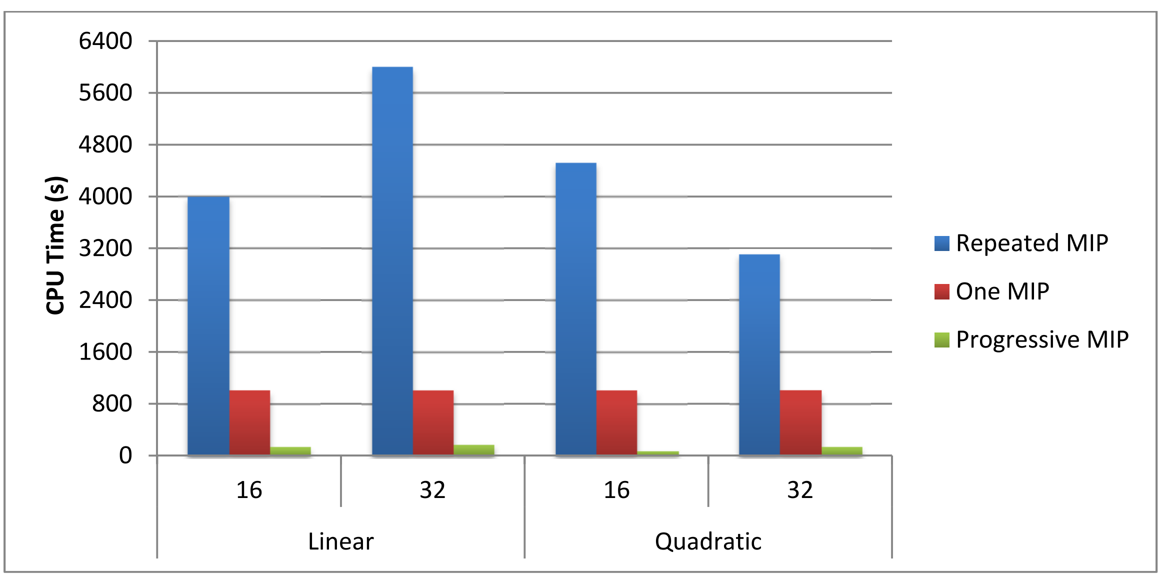


CPU Time

Table 30 reveals that once again, the progressive MIP was much faster than one MIP was much faster than the repeated MIP approach.

Table 30. 57 Bus, Tightly Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	4001.22	6003.74	4528.23	3115.72
One MIP	1015.75	1020.67	1009.96	1011.68
Progressive MIP	138.03	182.29	80.39	139.19



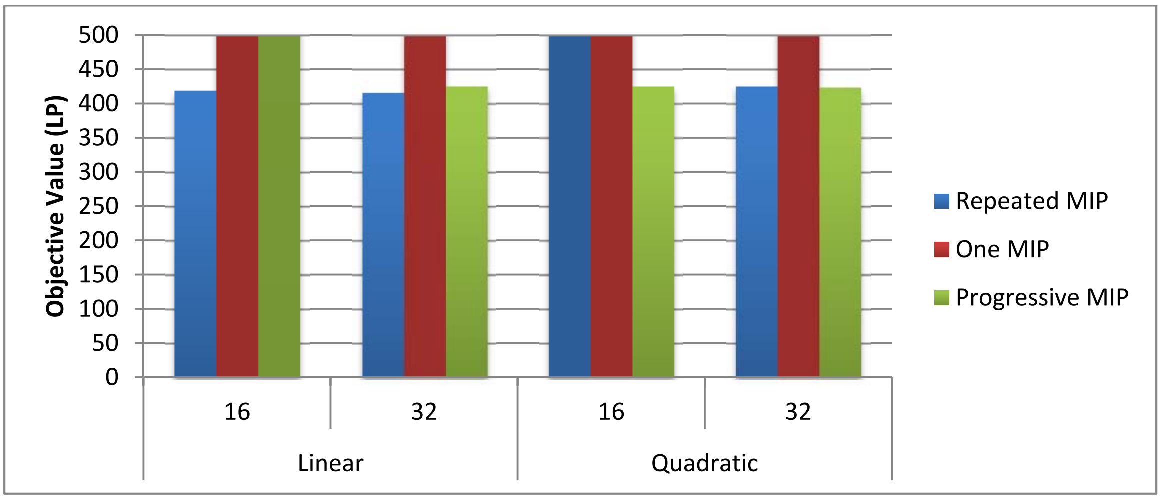
57 Bus, Loosely Constrained

Objective Value Function (LP)

Table 31 shows that 6 of the 12 approaches could not find a feasible solution to the loosely constrained 57 bus problem.

Table 31. 57 Bus, Loosely Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	419.36	416.56	INF	426.51
One MIP	INF	INF	INF	INF
Progressive MIP	INF	426.41	425.79	424.63
No lines open	422.6	422.4	423.3	423.5



LP vs. NLP Objective Function Value Differences

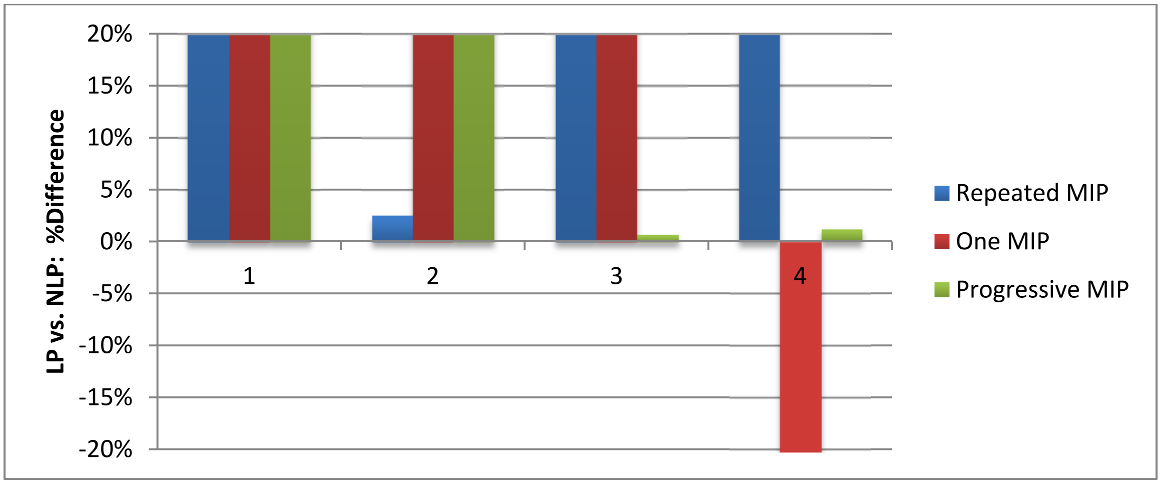
As seen in Tables 32 and 33, 9 of the 12 approaches could not find a feasible solution with the 57 bus problem with loose current constraint.

Table 32. 57 Bus, Loosely Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	INF	427.47	INF	INF
One MIP	INF	INF	INF	INF
Progressive MIP	INF	INF	428.93	430.18

Table 33. 57 Bus, Loosely Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	n/a	2.55	n/a	n/a
One MIP	n/a	n/a	n/a	n/a
Progressive MIP	n/a	n/a	0.73	1.29

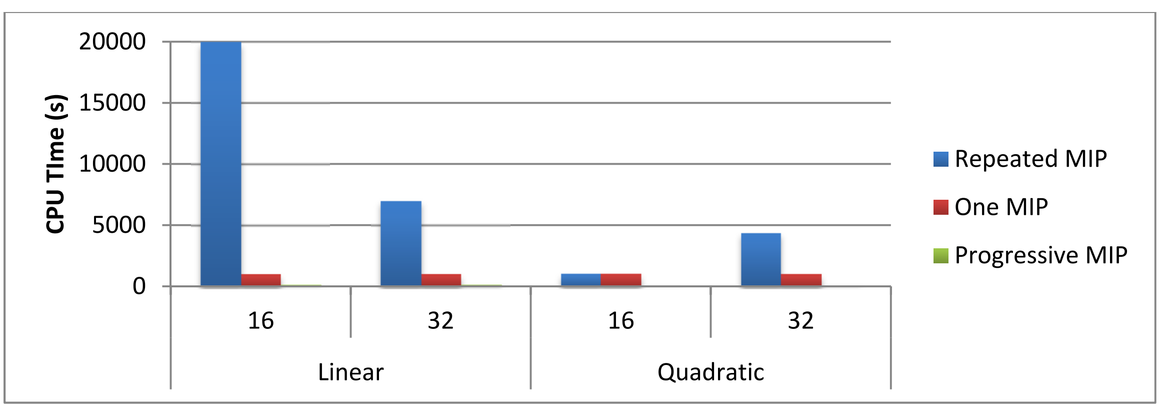


CPU Time

Table 34 shows that the progressive MIP is significantly faster than the one MIP approach, which is usually much faster than doing a repeated MIP. In this case, using the progressive MIP is both the fastest method and the most likely to find a feasible solution.

Table 34. 57 Bus, Loosely Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	20007.06	7005.45	1031.57	4388.51
One MIP	1016.32	1019.00	1010.00	1010.06
Progressive MIP	138.00	182.26	71.79	122.51



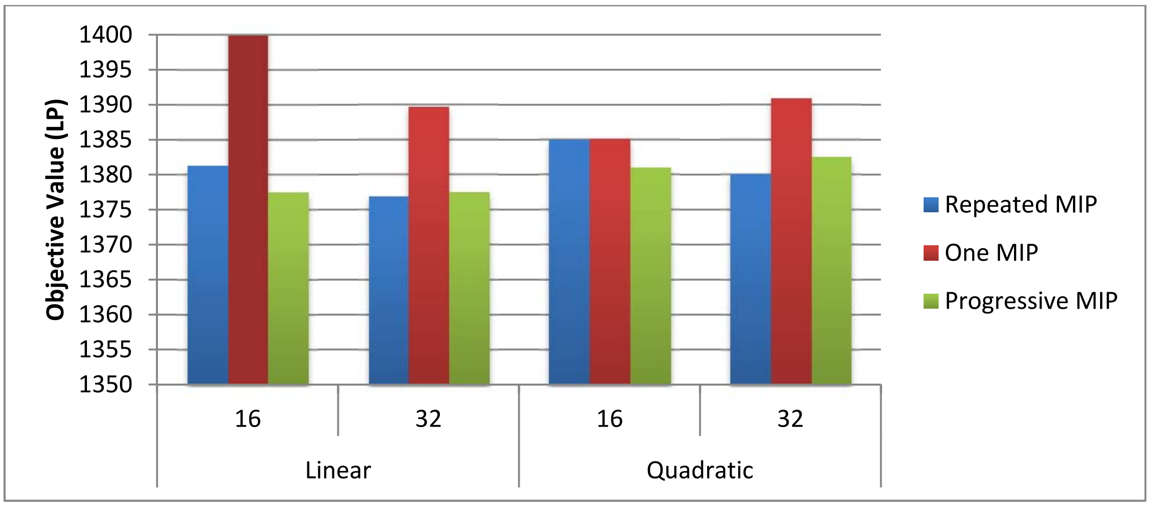
118 Bus, Tightly Constrained

Objective Value Function (LP)

As seen in Table 35, there is only one approach (one MIP, linear step size, 16 cuts) where the linear solver cannot find a feasible solution.

Table 35. 118 Bus, Tightly Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	1381.32	1377.03	1385.04	1380.17
One MIP	INF	1389.79	1385.13	1390.99
Progressive MIP	1377.49	1377.63	1381.14	1382.52
No lines open	1381.2	1380.4	1388.3	1388.3



LP vs. NLP Objective Function Value Differences

Tables 36 and 37 show that the NLP was able to find a feasible solution to the same configuration of lines as the LP where the LP approach could not find a feasible solution. In all other cases, the nonlinear solution was within 1% of the linear solution.

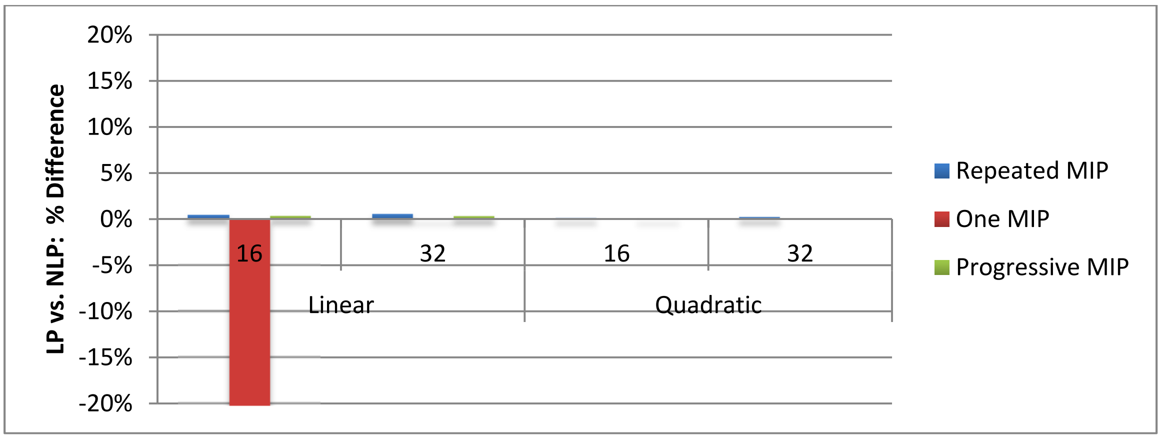
Table 36. 118 Bus, Tightly Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	1388.74	1386.37	1387.64	1384.41
One MIP	1385.19	1391.23	1385.19	1391.23

Progressive MIP	1383.35	1383.37	1382.15	1382.82

Table 37. 118 Bus, Tightly Constrained: % Difference between LP and NLP Objective Value

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	0.53	0.67	0.19	0.31
One MIP	INF	0.10	0.00	0.02
Progressive MIP	0.42	0.42	0.07	0.02

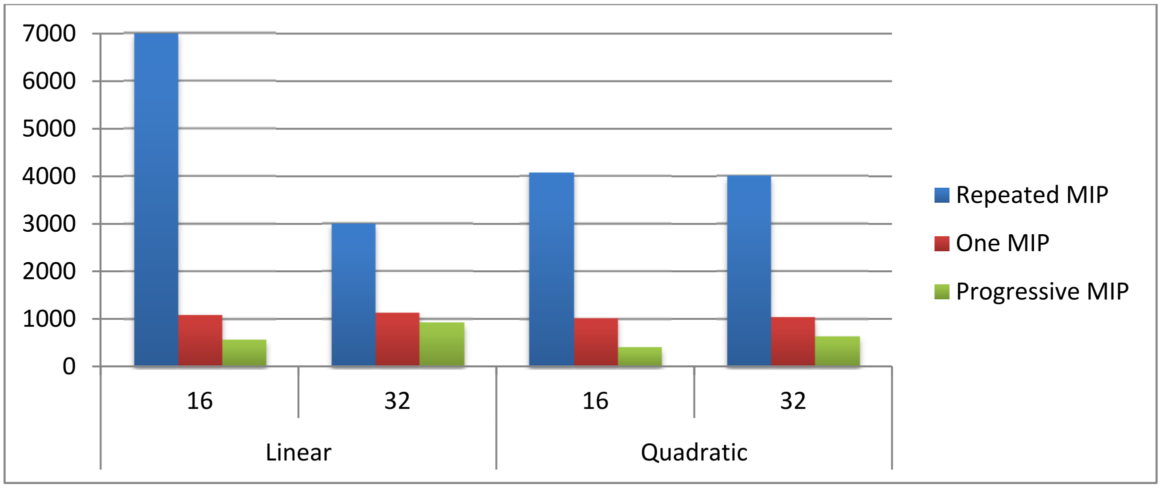


CPU Time

While the progressive MIP was faster than one MIP which was faster than the repeated MIP, the progressive MIP approach was comparatively much faster in the 57 bus cases than this case. Table 38 shows the progressive MIP speeding up solution times between 1.2 and 2.4x; in 57 bus case, the progressive MIP sped up solution times by at least 5x versus the one MIP approach.

Table 38. 118 Bus, Tightly Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	7008.08	3010.86	4089.92	4014.06
One MIP	1106.94	1149.24	1033.84	1043.54
Progressive MIP	591.78	936.41	422.64	652.28



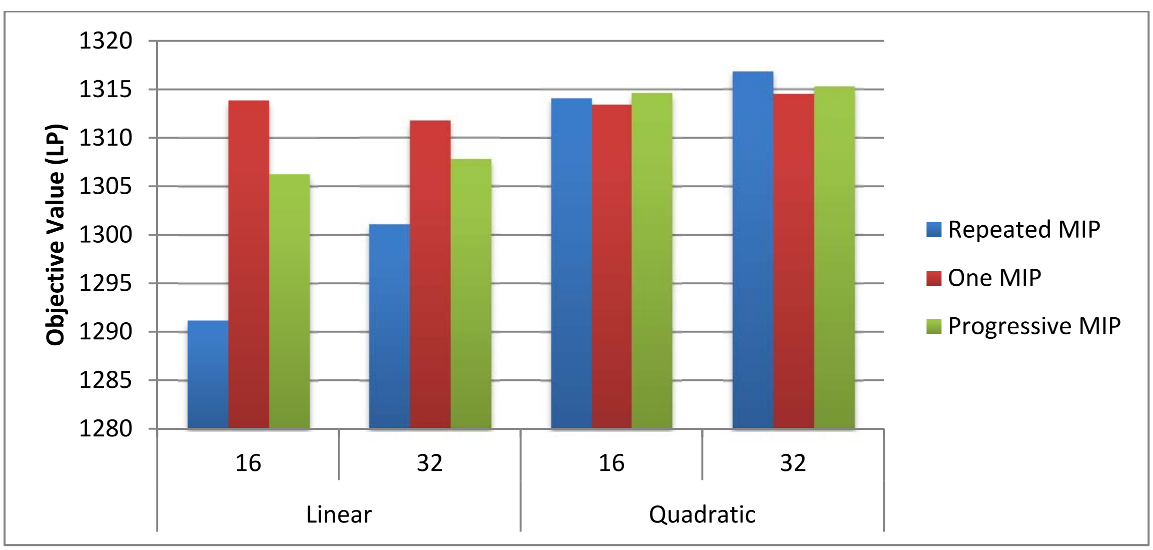
118 Bus, Loosely Constrained

Objective Value Function (LP)

As seen in Table 39, the 118 bus solutions mainly differ in the linear step size, repeated MIP approach versus all of the other approaches.

Table 39. 118 Bus, Loosely Constrained: Linear Objective Value (linear objective, linear constraints)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	1291.18	1301.09	1314.15	1316.88
One MIP	1313.85	1311.76	1313.47	1314.60
Progressive MIP	1306.34	1307.86	1314.62	1315.33
No lines open	1307.7	1311.8	1310.7	1314.6



LP vs. NLP Objective Function Value Differences

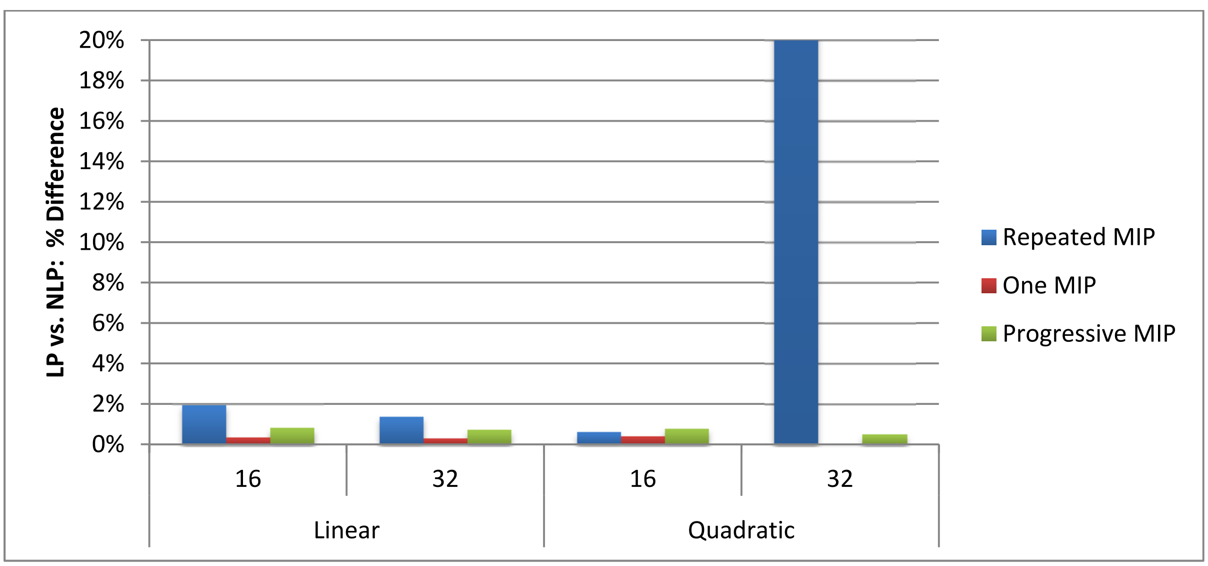
There was one line configuration out the 12 approaches where the nonlinear solver could not find a feasible solution, seen in Tables 40 and 41. Otherwise, the nonlinear answers were within 2% or less of the linear answers. Like in some of the other cases, the nonlinear solution for the configuration found with the repeated MIP, linear step size is more consistent with the other approaches than the linear solution. If we exclude the linear step size, repeated MIP approach and the infeasible answer, the nonlinear solution is within 1% of the linear solution.

Table 40. 118 Bus, Loosely Constrained: NLP Objective Value (nonlinear constraints, linear objective)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	1317.14	1319.32	1322.53	INF
One MIP	1318.49	1315.89	1318.77	1315.55
Progressive MIP	1317.29	1317.29	1325.20	1322.33

Table 41. 118 Bus, Loosely Constrained: % Difference between LP and NLP Objective Values

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	1.97	1.38	0.63	99.99
One MIP	0.35	0.31	0.40	0.07
Progressive MIP	0.83	0.72	0.80	0.53

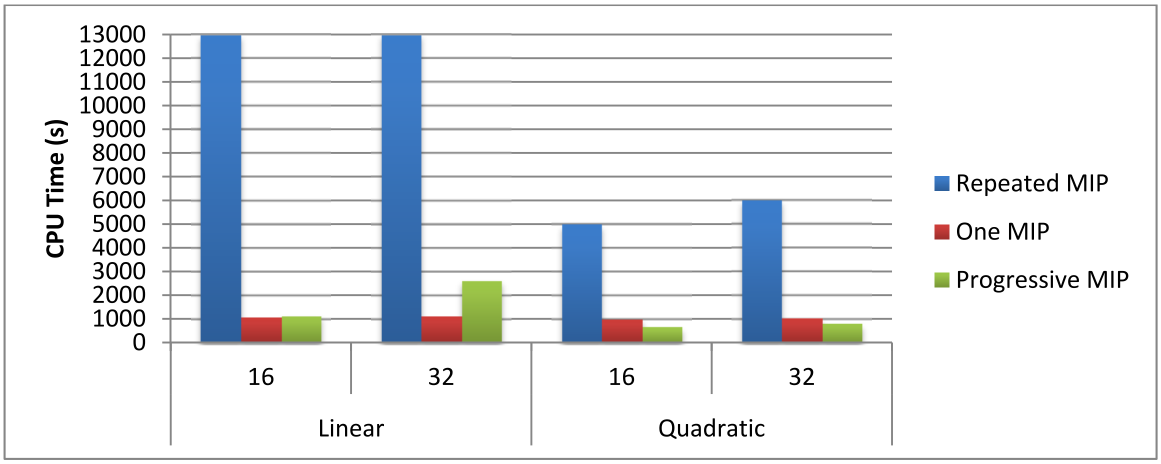


CPU Time

As displayed in Table 42, both the one MIP and progressive MIP approaches run much faster than the repeated MIP approaches; However, the progressive MIP is slower than the one MIP approach when the linear step size is used.

Table 42. 118 Bus, Loosely Constrained: CPU Time (seconds)

Step Size Function	Linear		Quadratic	
Number of Cuts	16	32	16	32
Repeated MIP	13009.89	13033.29	5003.83	6012.06
One MIP	1075.83	1134.47	1014.72	1038.70
Progressive MIP	1159.56	2642.57	666.49	818.48



6. Summary

From this study, we see that transmission switching provides cost benefits in AC systems. Tables 43 and 44 show that the magnitude of this cost savings is very dependent on the current constraints; one obtains more savings with lower line limits. We find that the linear approximation generally achieves about the same amount of savings as the original nonlinear program; however, there are cases where the linear program finds a better answer (like the 14 bus case with loose current limits) or a worse answer(like the 14 bus case with tight current limits). In this case, opening the first line provides the most of the benefits. The conflation of the MIP plus the changing fixed points occasionally leads to a better answer with allowing one line open rather than five lines open. This is often due to the five lines open problem choosing a network configuration that is better in the first iteration than opening one line but worse when the problem has finally converged.

Table 43. % Savings with up to one line open

Current Constraint	Method	14	30	57	118
Loose	Linear	3%	3%	0%	0%
	IPOPT	2%	4%	1%	0%
Tight	Linear	11%	3%	0%	0%
	IPOPT	13%	5%	3%	0%

Table 44. % Savings with up to five lines open

Current Constraint	Method	14	30	57	118
Loose	Linear	4%	2%	1%	1%
	IPOPT	2%	1%	0%	0%
Tight	Linear	10%	3%	1%	0%
	IPOPT	12%	3%	1%	0%

7. Conclusions

The one MIP approach solves the 14 and 30 bus problems quicker than the repeated MIP approach and within 4% or less of the repeated MIP objective value. However, while the one MIP approach has fast performance in the 57 and 118 bus problems, both the one and repeated MIP approaches find worse solutions than not switching any lines in many of the cases. In the larger networks, it appears that there are either multiple optimal solutions or multiple network configurations that have a very similar cost.

The linearized ACOPF is much faster than the unmodified ACOPF and usually finds solutions within 1% of the ACOPF. The progressive and one MIP approaches are much faster than the repeated MIP approach and generally give answers within 1% of the repeated MIP approach. However, there are instances where the linearized ACOPF finds an optimal network configuration that is not feasible in the unmodified ACOPF, due to the linearized ACOPF finding a solution with voltages that are below the minimum threshold.

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Appendix: Detailed Numerical Results

A.1 Full Repeated MIP versus MIP only on First Pass Results

14 bus problem.

Tables A.1 and A.2 show how the repeated MIP compares to the approach of using only one MIP. In the tightly constrained case, the objective values are very similar between both approaches, differing at the most by 0.25%. Using one MIP has a significant time advantage. The lines opened differ slightly, with the repeated MIP choosing to open two lines in some chases while the approach using one MIP only results in one line being opened.

The loosely constrained case displays more variability between the approaches using one MIP and the repeated MIP, although there is also more variability between step size/number of cuts approaches. However, the biggest difference in objective function values is 3.5%. The approach using one MIP is at least twice as fast as the repeated MIP approach. The lines open vary between one MIP and repeated MIP with the repeated MIP approach opening line 3-4 but never opening line 6-12, which the one MIP approach opens in every case. However, both cases open line 2-5, and the lines opened in the one MIP approach except for line 6-12 are opened in one of the repeated MIP approaches.

Table A.1. Repeated vs. One MIP: 14 Bus, Tight Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	No Lines Open	0	105.69	106.53	105.69	106.53
	Repeated MIP	5	95.02	95.04	95.10	95.04
	One MIP	5	95.25	95.04	95.25	95.04
CPU Time	Repeated MIP	5	34.7	101.0	26.7	59.1
	One MIP	5	8.4	20.6	15.8	22.6
No. of Lines Opened	Repeated MIP	5	1	1	1	1
	One MIP	5	2	1	2	1
Lines Opened	Repeated MIP	5	4-7	4-7	4-7	4-7
	One MIP	5	2-5 4-7	4-7	2-5 4-7	4-7

Table A.2: Repeated vs. One MIP: 14 Bus, Loose Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	No Open Lines	0	85.94	85.63	85.15	86.13
	Repeated MIP	5	82.53	82.10	84.65	84.38
	One MIP	5	84.49	85.08	84.49	85.08
CPU Time	Repeated MIP	5	258.6	152.4	126.0	165.3
	One MIP	5	22.1	59.4	23.1	40.2
No. of Lines Opened	Repeated MIP	5	2	5	5	5
	One MIP	5	5	5	5	5
Lines Opened	Repeated MIP	5	2-5 3-4	2-5 3-4 4-9 9-14 12-13	2-5 3-4 4-5 4-7 4-9	2-5 3-4 4-5 4-7 4-9
	One MIP		2-5 4-5 4-7 4-9 6-12	2-5 4-5 4-7 4-9 6-12	2-5 4-5 4-7 4-9 6-12	2-5 4-5 4-7 4-9 6-12

30 bus problem.

Table A.3 shows that the tight current constraint case has up to a 2.3% gap between the one and repeated MIP approaches. There is a clear advantage to opening lines. Using one MIP is at least twice as fast as using a MIP at every step. In both cases, line 6-28 is opened, but the other four lines are completely different in the case of linear and quadratic step size with16 cuts; 3 lines are different for the linear and quadratic step size with 32 cuts.

In Table A.4, the loose current constraint case has up to a 2% gap between the one and repeated MIP approaches. However, the one MIP approach always finds a more costly answer than the repeated MIP approach; in some cases, opening lines has a minor impact. Again, the one MIP approach is much faster than the repeated MIP approach. In both tight and loose cases, the quadratic step size with repeated MIP appears to run much faster than the linear step size. All cases open the line connecting buses 6 and 28; the other lines opened vary greatly between the one and repeated MIP approaches as well as the different step size and number of cuts approaches.

Table A.3: Repeated vs. One MIP: 30 Bus, Tight Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	Without Opening Lines	0	6.02	6.08	6.02	6.08
	Repeated MIP	5	5.83	5.83	5.96	5.93
	One MIP	5	5.94	5.97	5.96	5.97
CPU Time	Repeated MIP	5	3908.0	6886.7	1718.0	2409.3
	One MIP	5	763.2	1001.4	806.5	1001.0
# of Lines Opened	Repeated MIP	5	5	5	5	3
	One MIP	5	5	5	5	5
Lines Opened	Repeated MIP	5	6-28 9-11 10-21 10-22 23-24	6-28 9-11 10-21 10-22 16-17	6-28 9-11 10-21 10-22 23-24	6-28 9-11 25-27
	One MIP	5	6-28 10-20 12-15 14-15 25-27	6-28 10-20 12-15 16-17 25-27	6-28 10-20 12-15 14-15 25-27	6-28 10-20 12-15 16-17 25-27

Table A.4. Repeated vs. One MIP: 30 Bus, Loose Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	Without Opening Lines	0	5.96	5.98	5.96	5.98
	Repeated MIP	5	5.82	5.83	5.93	5.81
	One MIP	5	5.94	5.89	5.95	5.90
CPU Time	Repeated MIP	5	8834.5	7084.0	2871.7	1831.5
	One MIP	5	1000.5	1002.1	1000.5	1001.0
# of Lines Opened	Repeated MIP	5	5	5	5	4
	One MIP	5	5	4	5	4
Lines Opened	Repeated MIP	5	4-12 6-28 16-17 18-19 23-24	6-28 9-11 10-21 10-22 23-24	6-9 6-28 9-10 9-11 25-27	6-9 6-28 25-27 27-19
	One MIP	5	6-28 12-15 14-15 19-20 25-27	6-28 10-21 14-15 15-18	6-28 12-15 14-15 19-20 25-27	6-28 10-21 14-15 15-18

57 bus problem.

In the tightly constrained case, the repeated MIP finds a feasible solution while the one MIP approach does not, displayed in Table A.5. The lines chosen to open are very different between the two cases. In the both the tightly and loosely constrained cases (Table A.6), the one MIP approach does not find a feasible solution, while the repeated MIP finds a feasible solution in 3 of the 4 approaches. The reason the one MIP approach finds infeasible solutions is that the network configuration it chooses in the first iteration is infeasible, but in the first stage, more infeasible area is in the linear program that is cut off by iterative voltage constraints later.

Table A.5. Repeated vs. One MIP: 57 Bus, Tight Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	Without Opening Lines	0	424.02	422.71	424.1	423.7
	Repeated MIP	5	417.4	417.4	426.7	425.8
	One MIP	5	INF[1]	INF	INF	INF
CPU Time	Repeated MIP	5	4001	6003	4528	3116
	One MIP	5	10167	1021	1010	1012
# of Lines Opened	Repeated MIP	5	5	4	3	5
	One MIP	5	5	5	5	5
Lines Opened	Repeated MIP	5	1-15 2-3 11-13 13-15 53-54	3-4 3-15 22-38 53-54	9-10 9-13 11-13	2-3 3-4 3-15 22-38 54-55
	One MIP	5	9-10 9-55 11-43 14-46 35-36	9-13 10-51 13-49 14-46 38-44	9-10 9-55 11-43 14-46 35-36	9-13 10-51 13-49 14-46 38-44

Table A.6. Repeated vs. One MIP: 57 Bus, Loose Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	No Lines Open	0	422.6	422.4	423.3	423.5
	Repeated MIP	5	419.4	416.6	INF	426.5
	One MIP	5	INF	INF	INF	INF
CPU Time	Repeated MIP	5	20007	7006	1032	4389
	One MIP	5	1016	1019	1010	1010
No. of Lines Opened	Repeated MIP	5	5	5	5	5
	One MIP	5	5	5	5	5
Lines Opened	Repeated MIP	5	3-4 14-15 19-20 34-35 49-50	6-7 9-11 9-13 12-13 12-16	8-9 13-49 14-46 15-45 49-50	1-2 1-15 11-41 12-17 54-55
	One MIP	5	9-10 9-55 14-46 25-30 44-45	9-10 9-13 10-51 11-13 14-46	9-10 9-55 14-46 25-30 44-45	9-10 9-13 10-51 11-13 14-46

118 bus problem.

In the tightly constrained case (Table A.7), the repeated and one MIP approaches find solutions within 1% of each other if the case where the one MIP approach finds an infeasible solution is not included. The one MIP approach is more than 2.5 times faster than the repeated MIP approach. There is a large variance in which lines are selected to be open between the repeated and one MIP approaches, and between the different step size and number of cuts approaches. The one MIP approach finds higher costs solutions that the repeated MIP approach, and in two cases finds worse solutions than when no lines are open. In the loosely constrained case (Table A.8), feasible solutions are found in both repeated and one MIP approaches, and the answers are within 2% of each other. However, both the repeated MIP and one MIP approaches find some solutions that are worse than not opening any lines. The one MIP approach runs five times faster or more than the repeated MIP approach. The lines opened vary greatly between the different approaches, although some same lines show up in several different approaches.

Table A.7. Repeated vs. One MIP: 118 Bus, Tight Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	Without Opening Lines	0	1381.2	1380.4	1388.3	1388.3
	Repeated MIP	5	1381.3	1377.0	1385.0	1380.2
	One MIP	5	INF	1389.8	1385.1	1391
CPU Time	Repeated MIP	5	7008	3011	4090	4014
	One MIP	5	1107	1149	1034	1044
No. of Lines Opened	Repeated MIP	5	5	5	5	5
	One MIP	5	5	5	5	5
Lines Opened	Repeated MIP	5	60-61 61-62 61-64 75-77 77-82	5-6 6-7 47-49 48-49 77-80	34-36 35-36 42-49 77-82 82-83	42-49 60-62 61-62 92-94 93-94
	One MIP	5	32-113 77-82 92-94 92-100 109-110	8-30 47-49 48-49 92-93 94-100	32-113 77-82 92-94 92-100 109-110	8-30 47-49 48-49 92-93 94-100

Table A.8. Repeated vs. One MIP: 118 Bus, Loose Current Constraint

		Step Size Function	Linear		Quadratic	
		Number of Cuts	16	32	16	32
		<= # Lines Open				
Objective Value	Without Opening Lines	0	1307.7	1311.8	1310.7	1314.6
	Repeated MIP	5	1291.2	1301.1	1314.2	1316.9
	One MIP	5	1313.9	1311.8	1313.5	1314.6
CPU Time	Repeated MIP	5	13010	13033	5004	6012
	One MIP	5	1075.8	1134.5	1014.7	1038.7
# of Lines Opened	Repeated MIP	5	5	5	5	4
	One MIP	5	5	0	5	0
Lines Opened	Repeated MIP	5	1-2 8-30 11-12 65-68 94-100	1-2 54-55 64-65 76-118 80-99	41-42 42-49 49-66 68-81 110-112	65-68 82-83 99-100 110-112
	One MIP	5	61-64 75-118 77-80 92-102 100-103	n/a	61-64 75-118 77-80 92-102 100-103	n/a

A.2 PROGRESSIVE MIP

In the Tables that follow, the *Linear Obj Value* is the objective value of the linear iterative solver. The *NLP Obj Value* is the value of the nonlinear solution when the network is fixed as the optimal network found in the linear iterative solver. The *MIP gap* is the gap between the best-known linear solution and the final MIP answer. *New Opened Line* designates which line was opened in the problem. For example, the first problem only allows one line open. Then, the next problem fixes that line as open and sees if there is another line that would be beneficial to open, and so forth.

14 bus.

The linear and nonlinear solutions in the 14 bus case – both for the tight and loose constraints (A.9 and A.10) - are within 0.6% of each other except for one case where the linear solver found an infeasible solution. Like the repeated/one MIP approaches, the lines opened in the tight current constraint case were 4-7 and 2-5. The loose current constrained case consistently opens lines 2-5, 3-4, and 4-7, but differs on the 4^{th} and 5^{th} lines opened (and a 5^{th} line is only opened in one case). The linear solution was very close to the nonlinear solution for the corresponding network. The total CPU time for all the linear cases up to 5 lines open is close to the CPU time of using one MIP.

Table A.9. Progressive MIP: 14 Bus, Tight Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	95.05	95.07	95.06	95.04
	<=2	95.04	95.06	95.06	94.98
	<=3	95.33	95.07	95.41	94.99
	<=4	95.14	95.06	95.41	95.00
	<=5	95.19	INF	95.42	95.04
NLP Obj Value (IPOPT)	<=1	95.06	95.04	95.05	95.07
	<=2	95.06	95.00	95.03	95.06
	<=3	95.41	95.05	95.40	95.06
	<=4	95.41	94.99	95.28	95.06
	<=5	95.43	94.95	95.26	95.06
CPU Time	<=1	1.3	1.9	1.0	1.9
	<=2	1.1	1.6	0.7	1.4
	<=3	1.7	2.6	1.5	2.9
	<=4	1.4	4.1	1.7	3.7
	<=5	1.7	5.0	1.4	4.3
NLP CPU Time (IPOPT)	<=1	1.1	1.1	0.9	3.2
	<=2	1.3	1.3	0.9	1.0
	<=3	2.2	1.3	2.4	1.2
	<=4	4.5	1.6	2.1	2.1
	<=5	5.7	2.0	6.7	1.4
Linear Iterations	<=1	4	4	4	4
	<=2	4	3	4	3
	<=3	2	4	3	3
	<=4	3	3	4	2
	<=5	2	3	2	4
MIP Gap	<=1	1.5E-15	2.7E-15	1.5E-15	2.7E-15
	<=2	2.8E-04	9.5E-15	1.0E-14	8.4E-15
	<=3	4.4E-15	1.3E-05	3.9E-15	2.8E-14
	<=4	1.3E-14	9.5E-04	4.2E-04	9.6E-04
	<=5	2.2E-14	8.5E-04	1.4E-14	3.9E-14
New Opened Line	<=1	4-7	4-7	4-7	4-7
	<=2	None	None	None	None
	<=3	2-5	None	2-5	None
	<=4	None	None	None	None
	<=5	None	None	None	None

Table A.10. Progressive MIP: 14 Bus, Loose Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	85.17	85.17	85.28	85.61
	<=2	85.03	84.97	85.11	85.20
	<=3	85.04	85.08	85.03	85.09
	<=4	84.98	85.24	85.08	85.36
	<=5	85.01	85.21	85.09	85.36
NLP Obj Value (IPOPT)	<=1	85.61	85.61	85.61	85.29
	<=2	85.25	85.20	85.20	85.01
	<=3	85.10	85.10	85.09	85.10
	<=4	85.09	85.36	85.10	85.20
	<=5	85.02	85.36	85.10	85.23
CPU Time	<=1	0.9	1.3	0.9	1.2
	<=2	0.8	1.3	2.1	1.3
	<=3	0.8	1.0	2.0	1.0
	<=4	0.6	0.8	2.1	0.8
	<=5	1.2	0.7	2.6	0.8
NLP CPU Time (IPOPT)	<=1	1.0	1.0	0.8	1.2
	<=2	1.0	1.9	1.2	1.8
	<=3	3.4	4.1	0.7	1.7
	<=4	1.8	3.5	0.7	2.6
	<=5	3.2	4.8	0.6	1.4
Linear Iterations	<=1	6	6	4	4
	<=2	4	4	5	4
	<=3	3	3	3	3
	<=4	2	2	4	3
	<=5	3	3	3	2
MIP Gap	<=1	3.4E-15	8.5E-16	3.4E-15	8.5E-16
	<=2	3.0E-14	3.1E-15	2.8E-14	8.6E-16
	<=3	1.4E-15	1.0E-15	1.5E-15	3.6E-15
	<=4	4.6E-15	4.8E-15	2.9E-15	1.9E-14
	<=5	1.4E-15	3.7E-14	4.5E-15	1.1E-14
New Opened Line	<=1	2-5	2-5	2-5	2-5
	<=2	3-4	3-4	3-4	3-4
	<=3	4-7	4-7	4-7	4-7
	<=4	None	9-14	12-23	9-14
	<=5	4-5, 4-9	None	None	None

30 bus.

In the 30 bus case with tight current constraints (A.11), the linear objective value is within 3% of the objective value except for two cases – one where they differ by 12% and one where the solution is infeasible. With loose current constraints (A.12), the linear objective value is within 2% of the nonlinear objective value. The repeated and one MIP approaches share some of the same lines as opened by the progressive MIP approach, but the lines opened are not exactly the same between the approaches or within the different step size functions and number of cuts within the same approach. In the 30 bus problem, only one solution is found to be infeasible – when lines 6-28, 10-21, and 9-11 are opened in the tightly constrained case with quadratic step size, 32 cuts, and up to 4 lines open.

Table A.11. Progressive MIP: 30 Bus, Tight Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	5.92	5.93	5.92	5.93
	<=2	5.88	5.89	5.93	5.89
	<=3	5.89	5.90	5.94	5.89
	<=4	5.90	5.90	5.92	5.94
	<=5	5.89	5.90	5.93	5.93
NLP Obj Value (IPOPT)	<=1	5.93	6.10	5.97	6.76
	<=2	5.94	5.94	5.97	5.94
	<=3	5.94	5.96	5.94	5.94
	<=4	5.95	5.95	5.95	INF
	<=5	5.94	5.98	5.95	6.07
CPU Time	<=1	6.4	8.8	6.7	9.1
	<=2	5.4	7.7	3.6	6.5
	<=3	4.7	6.9	3.8	5.2
	<=4	4.6	5.5	3.3	19.1
	<=5	3.7	5.3	3.2	1.5
NLP CPU Time (IPOPT)	<=1	3.7	2.2	2.4	1.9
	<=2	11.8	8.2	7.2	6.0
	<=3	7.5	5.6	7.3	6.8
	<=4	4.8	4.6	6.6	2.1
	<=5	6.6	4.9	7.4	1.8
LinearIterations	<=1	4	3	4	3
	<=2	2	4	2	4
	<=3	4	2	4	2
	<=4	2	2	3	9

	<=5	2	2	3	2
MIP Gap	<=1	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=2	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=3	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=4	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=5	1.8E-07	1.7E-07	1.7E-07	7.6E-09
New Opened Line	<=1	6-28	6-28	6-28	6-28
	<=2	10-21	10-21	10-17	10-21
	<=3	14-15	15-18	25-27	None
	<=4	15-18	14-15	18-19	9-11
	<=5	12-15	None	12-14	15-18

Table A.12. Progressive MIP: 30 Bus, Loose Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	5.92	5.92	5.92	5.92
	<=2	5.92	5.92	5.93	5.92
	<=3	5.90	5.89	5.89	5.89
	<=4	5.90	5.90	5.90	5.90
	<=5	5.89	5.90	5.90	5.91
NLP Obj Value (IPOPT)	<=1	5.94	5.92	5.93	6.04
	<=2	5.93	5.92	5.94	5.92
	<=3	5.94	5.94	5.96	5.94
	<=4	5.96	5.95	5.95	5.97
	<=5	5.98	5.95	5.95	5.98
CPU Time	<=1	4.6	5.5	4.5	6.2
	<=2	5.5	7.7	3.5	6.4
	<=3	4.8	6.0	4.5	6.2
	<=4	5.1	5.7	3.7	5.5
	<=5	5.9	4.8	3.9	4.5
NLP CPU Time (IPOPT)	<=1	6.8	6.6	8.5	2.8
	<=2	3.7	3.8	8.9	4.0
	<=3	18.5	3.2	8.5	2.8
	<=4	3.5	6.5	5.3	11.7
	<=5	5.7	7.6	8.1	3.6
LinearIterations	<=1	4	2	4	2
	<=2	2	2	2	2
	<=3	4	2	4	2
	<=4	10	2	4	2
	<=5	3	2	4	2

MIP Gap	<=1	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=2	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=3	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=4	1.7E-07	1.7E-07	1.7E-07	1.7E-07
	<=5	1.7E-07	1.7E-07	1.7E-07	1.7E-07
New Opened Line	<=1	6-28	24-25	6-28	24-25
	<=2	25-27	6-28	19-20	6-28
	<=3	10-21	10-21	10-21	10-21
	<=4		10-20	14-15	10-20
	<=5	27-30, 18-19			12-14

57 bus.

The 57 bus problem (A.13 and A.14) exhibits some peculiarities. There are many cases where the linear iterative solver finds what it believes to be a feasible solution, but the nonlinear solver reveals that the configuration is not feasible (usually due to violating the minimum voltage constraint). Out of the 20 different tests performed for the tight current case, only 8 find a feasible solution. However, this approach is much faster than even the one MIP approach and does not perform any worse than the one MIP approach. The tightly constrained problem appears to be very sensitive to what approach is used. The first three problems, where up to 1, then 2, then 3 lines are allowed open all have the same network configuration; however, the nonlinear solver finds some solutions that are feasible and some that are infeasible for the 2 and 3 lines open cases. This is likely due to different starting points being used in the nonlinear solver. In addition, it is surprising that some of the approaches recommend opening additional lines even when it appears to increase the total cost (see the linear step size function with 32 cuts; each additional line open increases the cost, although the solver has the option not to open any additional lines0.

In the loose current case, only 14 of the 20 tests find a feasible solution. The loosely constrained case also appears to be sensitive to what step size and number of cuts approach is used. It also opens the same three lines in all approaches, but for up to 2 or 3 lines open, the nonlinear solver can find a feasible solution with the given configuration in some approaches and not in others, likely due to the starting point used. In addition, the linear approach also opens lines in some cases when it increases the total cost (see the quadratic step size function with 16 cuts, for example).

Table A.13. Progressive MIP: 57 Bus, Tight Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	423.10	421.42	423.95	424.04
	<=2	423.18	422.85	422.79	422.84
	<=3	422.14	424.44	423.66	424.45
	<=4	427.76	424.84	425.20	423.44
	<=5	INF	428.56	426.15	427.96
NLP Obj Value (IPOPT)	<=1	429.56	429.56	429.56	429.56
	<=2	INF	INF	430.28	430.30
	<=3	427.43	INF	INF	INF
	<=4	INF	INF	INF	426.17
	<=5	INF	INF	427.62	INF
CPU Time	<=1	37.1	60.1	35.6	52.4
	<=2	20.5	36.5	9.4	12.7
	<=3	21.7	38.2	14.8	28.3
	<=4	32.3	15.8	12.1	17.5
	<=5	26.4	31.6	8.4	28.3
NLP CPU Time (IPOPT)	<=1	29.4	28.2	15.9	27.4
	<=2	86.9	26.6	33.3	25.5
	<=3	49.3	248.3	40.2	51.1
	<=4	7.0	32.6	12.7	48.7
	<=5	7.3	62.4	32.1	9.8
Linear Iterations	<=1	4	7	4	4
	<=2	4	10	3	3
	<=3	4	20	4	5
	<=4	20	3	5	3
	<=5	20	3	3	20
MIP Gap	<=1	2.5E-15	5.1E-14	2.5E-15	5.1E-14
	<=2	1.5E-14	1.4E-14	7.3E-04	1.0E-03
	<=3	2.0E-14	1.9E-15	9.6E-16	1.9E-15
	<=4	9.6E-16	3.3E-15	3.6E-15	4.7E-04
	<=5	3.4E-13	1.9E-15	7.9E-04	6.8E-16
New Opened Line	<=1	3-4	3-4	3-4	3-4
	<=2	54-55	54-55	53-54	53-54
	<=3	22-38	22-38	22-38	22-38
	<=4	32-34	1-15	1-15	14-15
	<=5	20-21	3-15	12-13	13-14

Table A.14. Progressive MIP: 57 Bus, Loose Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	421.36	422.93	423.09	423.27
	<=2	422.19	422.92	423.42	422.75
	<=3	421.99	421.78	423.61	423.34
	<=4	428.17	422.95	424.81	424.44
	<=5	INF	426.41	425.79	424.63
NLP Obj Value (IPOPT)	<=1	425.14	425.14	425.14	425.14
	<=2	426.43	426.43	426.44	425.10
	<=3	426.21	INF	426.51	INF
	<=4	INF	432.88	INF	426.55
	<=5	INF	INF	428.93	430.18
CPU Time	<=1	33.1	48.0	31.3	48.7
	<=2	29.2	26.8	8.6	14.1
	<=3	18.2	37.1	13.3	23.5
	<=4	31.5	32.1	10.8	17.2
	<=5	26.1	38.3	7.8	19.1
NLP CPU Time (IPOPT)	<=1	26.8	19.9	21.6	27.5
	<=2	53.0	22.2	53.1	29.3
	<=3	28.8	19.6	39.5	79.5
	<=4	19.0	47.9	10.6	27.0
	<=5	11.8	17.9	70.3	59.3
Linear Iterations	<=1	4	4	4	4
	<=2	10	3	4	3
	<=3	6	10	4	4
	<=4	20	11	5	3
	<=5	20	20	3	3
MIP Gap	<=1	9.4E-15	4.9E-13	9.4E-15	4.9E-13
	<=2	1.4E-14	6.0E-15	7.4E-04	7.4E-04
	<=3	2.7E-16	5.7E-15	5.1E-15	3.6E-15
	<=4	1.8E-15	1.8E-15	0.0E+00	2.7E-15
	<=5	1.4E-12	9.5E-04	8.1E-04	4.1E-16
New Opened Line	<=1	3-4	3-4	3-4	3-4
	<=2	54-55	54-55	54-55	54-55
	<=3	22-38	22-38	22-38	22-38
	<=4	32-34	14-15	1-15	1-15
	<=5	21-22	13-14	12-13	9-11

118 bus.

In the tightly constrained problem (A.15), the linear objective value is within 1% or less of the nonlinear objective value except for one case (up to 2 lines open, 32 cuts, quadratic step size). The first line opened is the same in all cases. The 2nd, 3rd, and 4th lines opened are consistent within the same step-size approach. However, the nonlinear program cannot find a feasible solution in the 2 lines open, 32 cuts, quadratic step size case while it can find a feasible solution in the 2 lines open, 16 cuts, quadratic step size case with the same network configuration; this difference is likely due to the difference in starting points.

In the loosely constrained problem (A.16), the difference between the objective value the linear iterative program found and the nonlinear program found is less than 1%, and there were not any infeasible solutions. The first line opened is consistent in all approaches, while the other lines that are opened differ between approaches.

Table A.15. Progressive MIP: 57 Bus, Tight Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	1379.12	1378.16	1385.68	1385.79
	<=2	1377.59	1378.33	1385.31	1384.70
	<=3	1377.37	1376.77	1381.49	1383.92
	<=4	1376.41	1376.80	1382.34	1382.95
	<=5	1377.49	1377.63	1381.14	1382.52
NLP Obj Value (IPOPT)	<=1	1385.82	1385.82	1385.82	1385.97
	<=2	1384.13	1384.13	1385.63	INF
	<=3	1383.26	1383.27	1384.40	1383.95
	<=4	1383.05	1383.05	1383.15	1383.63
	<=5	1383.35	1383.37	1382.15	1382.82
CPU Time	<=1	182.7	279.3	186.3	339.5
	<=2	108.0	181.2	26.6	46.0
	<=3	101.3	183.9	82.9	125.4
	<=4	106.6	161.6	65.9	40.8
	<=5	93.2	130.5	61.0	100.5
NLP CPU Time (IPOPT)	<=1	174.1	8.0	101.4	54.6
	<=2	213.6	11.5	148.8	235.9
	<=3	142.4	9.7	107.0	167.4
	<=4	127.1	10.4	139.6	119.3
	<=5	125.5	8.0	178.0	125.5

Linear Iterations	<=1	3	3	9	15
	<=2	3	3	2	4
	<=3	3	3	3	15
	<=4	3	3	5	4
	<=5	3	3	6	6
MIP Gap	<=1	7.2E-15	3.0E-14	7.2E-15	3.0E-14
	<=2	9.9E-04	7.0E-04	8.4E-04	1.0E-03
	<=3	7.4E-04	9.2E-04	3.4E-04	7.5E-04
	<=4	5.9E-04	4.5E-04	9.3E-04	8.2E-04
	<=5	8.8E-04	9.1E-05	9.8E-04	8.8E-04
New Opened Line	<=1	77-82	77-82	77-82	77-82
	<=2	82-96	82-96	60-62	60-62
	<=3	47-49	47-49	77-82	77-82
	<=4	48-49	48-49	82-96	82-96
	<=5	45-49	45-49	61-62	93-94

Table A.16. Progressive MIP: 118 Bus, Loose Current Constraint

	Step Size Function	Linear		Quadratic	
	Number of Cuts	16	32	16	32
	<= Number of Lines Open				
Obj Value	<=1	1310.10	1309.91	1315.61	1313.40
	<=2	1309.08	1310.11	1315.14	1315.23
	<=3	1308.54	1311.02	1316.49	1312.51
	<=4	1310.71	1308.90	1312.58	1318.96
	<=5	1306.34	1307.86	1314.62	1315.33
NLP Obj Value (IPOPT)	<=1	1316.32	1316.32	1316.42	1316.38
	<=2	1317.05	1317.11	1321.49	1317.02
	<=3	1317.17	1317.32	1317.02	1316.68
	<=4	1317.06	1317.43	1317.26	1320.89
	<=5	1317.29	1317.29	1325.20	1322.33
CPU Time	<=1	470	1597	437	1506
	<=2	192	303	42	74
	<=3	173	247	54	50
	<=4	201	250	34	336
	<=5	123	245	99	88
NLP CPU Time (IPOPT)	<=1	168	120	91	136
	<=2	130	120	179	155
	<=3	115	119	149	254
	<=4	232	177	123	101
	<=5	200	117	171	173
Linear Iterations	<=1	11	11	7	5

	<=2	11	13	6	6
	<=3	10	11	8	2
	<=4	11	10	4	5
	<=5	2	9	5	2
MIP Gap	<=1	3.9E-14	4.0E-14	3.9E-14	4.0E-14
	<=2	9.7E-04	2.2E-11	1.0E-03	7.3E-04
	<=3	2.2E-11	2.2E-11	9.8E-04	8.5E-04
	<=4	4.3E-09	3.3E-04	9.6E-04	4.4E-04
	<=5	2.2E-11	7.8E-04	8.4E-04	4.0E-04
New Opened Line	<=1	80-81	80-81	80-81	80-81
	<=2	69-77	69-77	8-30	61-64
	<=3	61-64	23-24	94-100	23-24
	<=4	76-118	61-64	47-49	59-63
	<=5	75-77	76-118	65-68	94-100