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# A “SuperOPF” Framework

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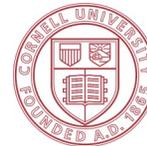
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**CERTS**  
CONSORTIUM FOR ELECTRIC RELIABILITY TECHNOLOGY SOLUTIONS

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# Background Motivation

The **ability** to optimally **allocate** and properly **value** system resources, including reliability, is needed for ...

- design and operation of electricity **markets** for energy, capacity, ancillary services, on all time scales from real-time to multi-year forward markets
- power grid **operations**: unit commitment, dispatch, maintenance
- **regulatory oversight**: market monitoring, reliability standards, impacts of environmental regulation
- resource **planning**: optimal investment, reliability studies, economic and reliability impacts of changes in technology (wind, solar, PHEV, DER, CHP, smart grid)

# Limitations of Current Practice

- Problem broken into sequential sub-problems
- DC models with proxies for AC constraints
  - misleading prices, especially for stressed system
- Stressed conditions are exactly when correct prices are most informative for identifying ...
  - location of existing network weaknesses
  - new equipment needed to upgrade network
  - net economic benefits of upgrades

# Extend Traditional OPF

- consistent framework for scheduling on multiple time scales
- handling of different levels of uncertainty
  - increased penetration of uncertain sources (wind, solar)
- new technologies
  - increased storage, smart grid applications (smart charging)
- proper valuation of resources, including reliability
  - proxy constraints can result in misleading prices
  - especially important under stressed conditions

# General Approach

Power systems operations and planning problems are computationally **very** complex.

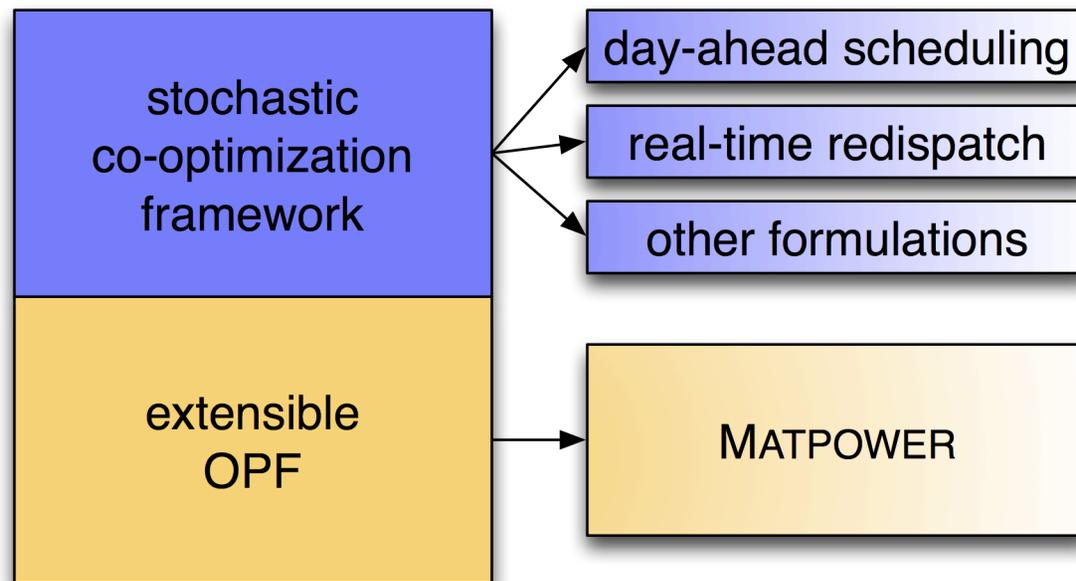
Traditional Approach	SuperOPF
Break into manageable sub-problems.	Combine into single mathematical programming framework.
DC network approximations	full AC network model
sequential optimization using proxy constraints	simultaneous co-optimization with explicit contingencies
misleading prices	more accurate prices

# Problems Combined

Combines several standard problems found in system operation & planning into single mathematical programming framework

- standard OPF with **full AC non-linear network model** & constraints
- **$n-1$  contingency security** with **static** (post-contingency voltage and flow limits) and **dynamic** (generator ramp limits, voltage angle difference limits) constraints
- procurement of adequate supply of **active & reactive energy** and corresponding **geographically distributed reserves**
- **uncertainty** of demand, wind, contingencies
- **stochastic cost**, including cost of post-contingency states
- explicit valuation of reliability through **cost of involuntary load shedding**
- **more accurate prices** for day-ahead contracts for energy, reactive supply, reserves
- consistent mechanism for subsequent **redispatch and pricing**, given specific realization of uncertain quantities

# Two Level Structure



# Co-optimization Structure



$$\min_{\Theta, V, P, Q} \sum_{i=1}^{n_g} [f_P^i(p_i) + f_Q^i(q_i)]$$

subject to

$$g_P(\Theta, V, P) = 0$$

$$g_Q(\Theta, V, Q) = 0$$

$$h_f(\Theta, V) \leq 0$$

$$h_t(\Theta, V) \leq 0$$

$$\theta_{\text{ref}} \leq \theta_i \leq \theta_{\text{ref}},$$

$$v_i^{\min} \leq v_i \leq v_i^{\max},$$

$$p_i^{\min} \leq p_i \leq p_i^{\max},$$

$$q_i^{\min} \leq q_i \leq q_i^{\max},$$

$$i = i_{\text{ref}}$$

$$i = 1 \dots n_b$$

$$i = 1 \dots n_g$$

$$i = 1 \dots n_g$$

- power flow scenario
  - all standard OPF variables, constraints, costs
  - “high probability”, “base” or “intact” case

# Co-optimization Structure



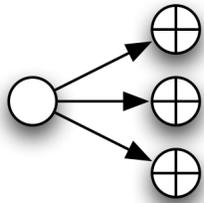
- power flow scenario
  - all standard OPF variables, constraints, costs
  - “low probability”, “contingency” case

# Co-optimization Structure



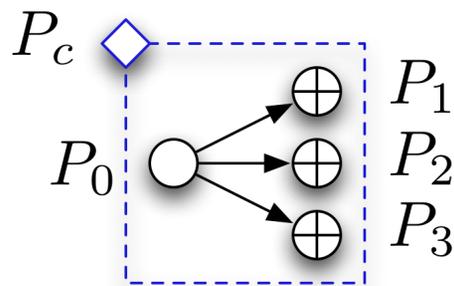
- transition constraint (e.g. ramp rate)
- restricts deviations of dispatch in contingency from dispatch in base case
- costs of scenarios are weighted (e.g. by probability)

# Co-optimization Structure



- base case and set of contingencies with
  - ramp limits constraining transitions from the base case to contingency cases
  - probability weighted costs

# Co-optimization Structure

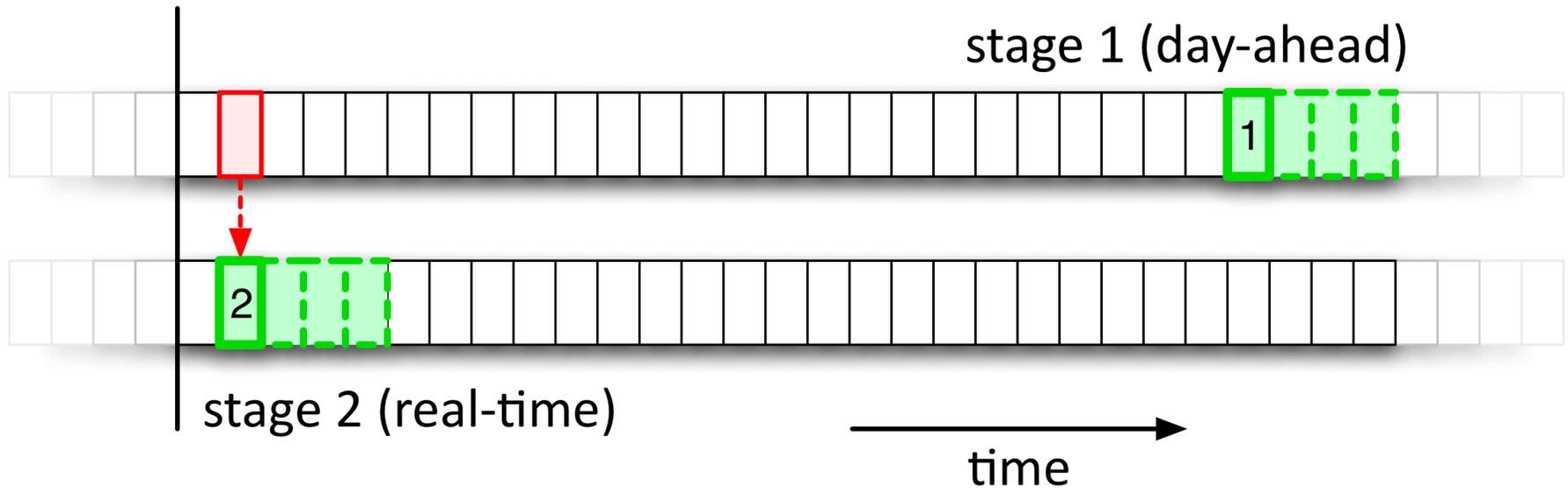


- root variable set with
  - upward and downward deviation variables
  - deviation limit variables
  - costs on deviations
  - costs and constraints on limits
- e.g. optimal energy contract, incs/decs from the contract, reserves

# Additional Comments on Structure

- contingency selection is important
- not simple Monte Carlo sampling
  - typical Monte Carlo misses low probability tails
- limits are hard limits that define an envelope that captures all operating points
  - still conservative
  - samples high probability cases sparsely
  - oversamples low probability cases
- expected prices
  - note: these are not probability weighted LMPs

# Two Stage, Receding Horizon



## Stage 1

- higher uncertainty
- make availability decisions
- set up financial contracts

## Stage 2

- some financial positions already taken
- some availability commitments already made
- decreased resource flexibility
- decreased uncertainty about system conditions
- updated system conditions
- possible additional resources available
- possible updated bids/offers

# Notation

$n_c$	number of contingencies
$n_g$	total number of dispatchable units (gens or loads)
$k$	index for contingencies (0 for base case)
$i$	index for dispatchable units (gens or loads)
$p_{ik}$	real power output for unit $i$ in contingency $k$
$p_{ci}$	day-ahead contracted real power output for unit $i$
$p_{ik}^+, p_{ik}^-$	upward, downward deviations of $p_{ik}$ from $p_{ci}$
$r_{Pi}^+, r_{Pi}^-$	upward, downward reserves ( $\max_k p_{ik}^+, \max_k p_{ik}^-$ ) for unit $i$
$\pi_k$	probability of contingency $k$
$C(\cdot)$	cost function
$G^k$	set of units available in contingency $k$
$R_{Pi}^{\max+}, R_{Pi}^{\max-}$	upward, downward reserve capacity limits for unit $i$
$\Delta_{Pi}^+, \Delta_{Pi}^-$	upward, downward physical ramp limits for unit $i$

replace  $p$  with  $q$  and  $P$  with  $Q$  for corresponding values for reactive power

# Stage 1 Objective Function

$$\min_{\substack{\Theta, V, P, Q, \\ P_c, P^+, P^-, \\ R_P^+, R_P^-}} \left\{ \sum_{k=0}^{n_c} \pi_k \sum_{i \in G^k} C_{Pi}(p_{ik}) + C_{Pi}^+(p_{ik}^+) + C_{Pi}^-(p_{ik}^-) \right. \\ \left. + \sum_{i=1}^{n_g} C_{RPi}^+(r_{Pi}^+) + C_{RPi}^-(r_{Pi}^-) \right\}$$

plus corresponding terms for reactive power

# Standard OPF Constraints

... replicated for each contingency

- nodal real & reactive power balance equations
- branch flow limits, voltage limits, generation limits, etc.

$$\left. \begin{aligned} g_P^k(\theta^k, V^k, P^k, Q^k) &= 0 \\ g_Q^k(\theta^k, V^k, P^k, Q^k) &= 0 \\ h^k(\theta^k, V^k, P^k, Q^k) &\leq 0 \end{aligned} \right\} k = 0 \dots n_c$$

# Stage 1 Linking Constraints

$$\left. \begin{aligned} 0 &\leq p_{ik}^+ \\ p_{ik} - p_{ci} &\leq p_{ik}^+ \\ p_{ik}^+ &\leq r_{Pi}^+ \leq R_{Pi}^{\max+} \end{aligned} \right\}$$

$$\forall k, \forall i \in G_k$$

$$\left. \begin{aligned} 0 &\leq p_{ik}^- \\ p_{ci} - p_{ik} &\leq p_{ik}^- \\ p_{ik}^- &\leq r_{Pi}^- \leq R_{Pi}^{\max-} \end{aligned} \right\}$$

$$-\Delta_{Pi}^- \leq p_{ik} - p_{i0} \leq \Delta_{Pi}^+$$

$$k = 1 \dots n_c, \forall i \in G_k$$

$$-\alpha \leq p_{i0} - p_{ci} \leq \alpha$$

$$\forall i, \alpha \in \{0, \infty\}$$

plus corresponding constraints for reactive power

# Stage 2 : Real-time Redispatch

- Same as day-head except:
  - updated scenarios (demand forecasts, available equipment, wind forecasts, credible contingencies, probabilities, etc.)
  - redispatch is relative to the now fixed contract from stage 1
  - reserve quantities from stage 1 appear as fixed limits on redispatch
- Implemented two formulations to simulate the two types of realized scenarios
  1. base or “intact” scenario
    - continue to guard against contingencies
  2. contingency or “outage” scenario
    - becomes new base case, temporarily ignore possibility of further contingencies (did not plan for  $n-2$ ).

# Stage 2 Objective Function

$$\min_{\substack{\Theta, V, P, Q, \\ P^+, P^-}} \left\{ \sum_{k=0}^{n_c} \pi_k \sum_{i \in G^k} C_{Pi}(p_{ik}) + C_{Pi}^+(p_{ik}^+) + C_{Pi}^-(p_{ik}^-) \right\}$$

plus corresponding terms for reactive power

# Standard OPF Constraints

... replicated for each contingency

- nodal real & reactive power balance equations
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$$\left. \begin{aligned} g_P^k(\theta^k, V^k, P^k, Q^k) &= 0 \\ g_Q^k(\theta^k, V^k, P^k, Q^k) &= 0 \\ h^k(\theta^k, V^k, P^k, Q^k) &\leq 0 \end{aligned} \right\} k = 0 \dots n_c$$

# Stage 2 Linking Constraints

$$\left. \begin{array}{l} 0 \leq p_{ik}^+ \\ p_{ik} - \hat{p}_{ci} \leq p_{ik}^+ \\ p_{ik}^+ \leq \hat{r}_{Pi}^+ \\ \\ 0 \leq p_{ik}^- \\ \hat{p}_{ci} - p_{ik} \leq p_{ik}^- \\ p_{ik}^- \leq \hat{r}_{Pi}^- \end{array} \right\}$$

$$\forall k, \forall i \in G_k$$

$$-\Delta_{Pi}^- \leq p_{ik} - p_{i0} \leq \Delta_{Pi}^+$$

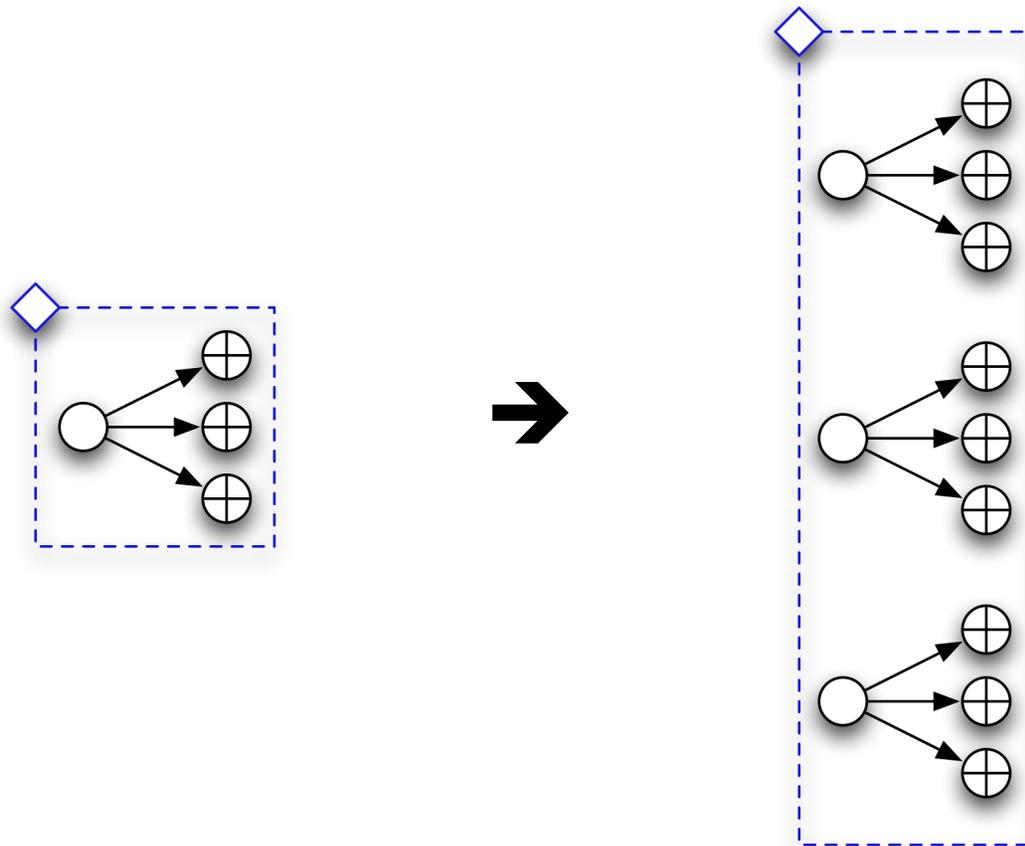
$$k = 1 \dots n_c, \forall i \in G_k$$

plus corresponding constraints for reactive power

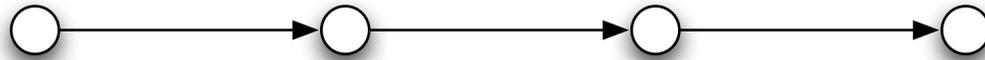
# New Formulations

- multiple “base” cases for improved modeling of sources with uncertainty (e.g. wind)
- multiple time periods
- intertemporal storage constraints
- discrete decision variables (unit commitment)
- generation and transmission investment

# Sources with Uncertainty (Wind)

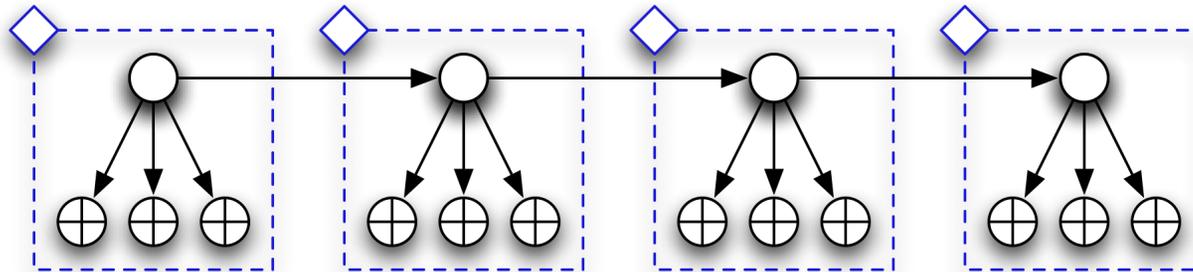


# Multi-period Optimization

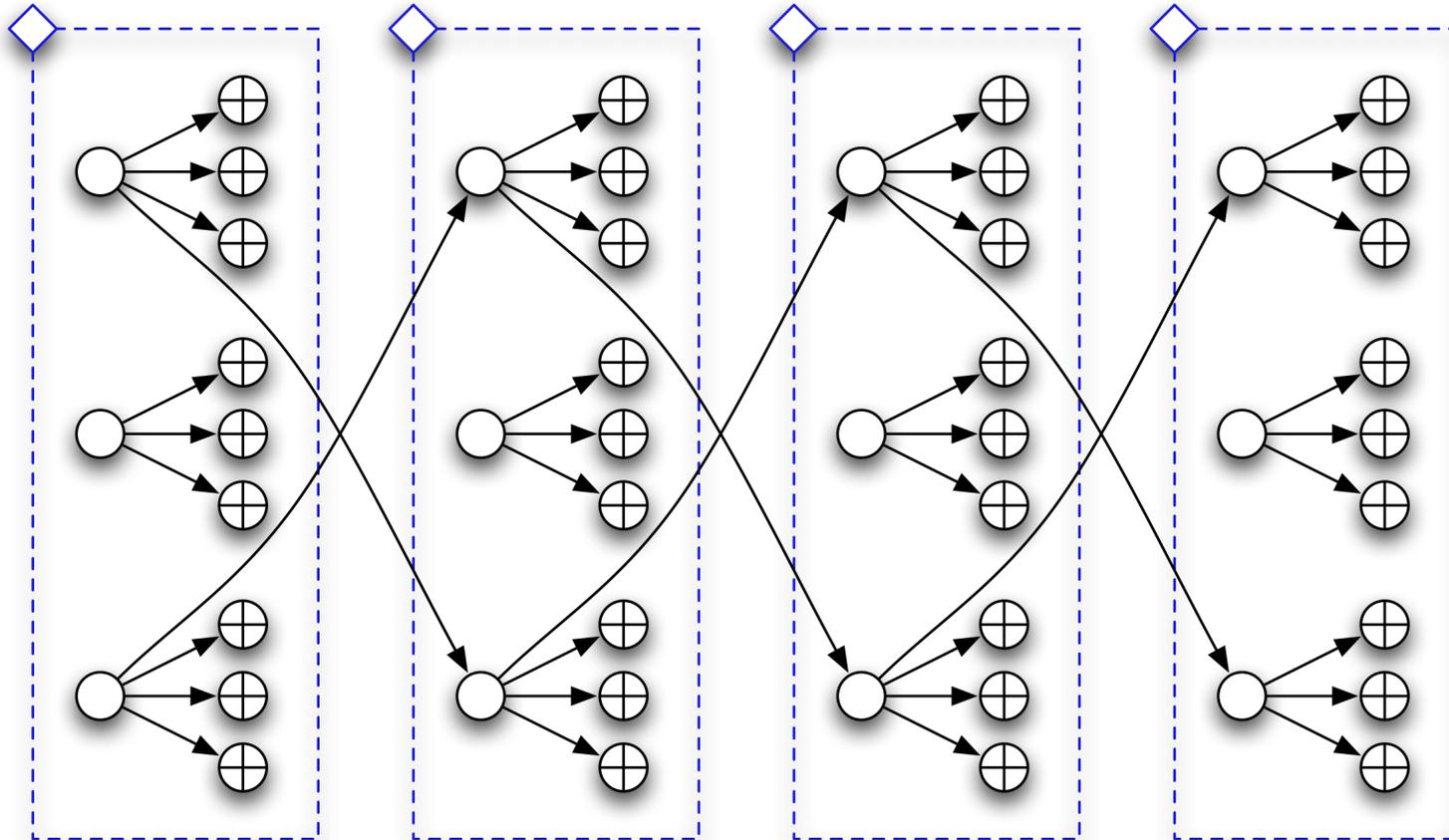


- independent OPFs, plus
- linking ramp constraints and costs

# Multi-period Optimization

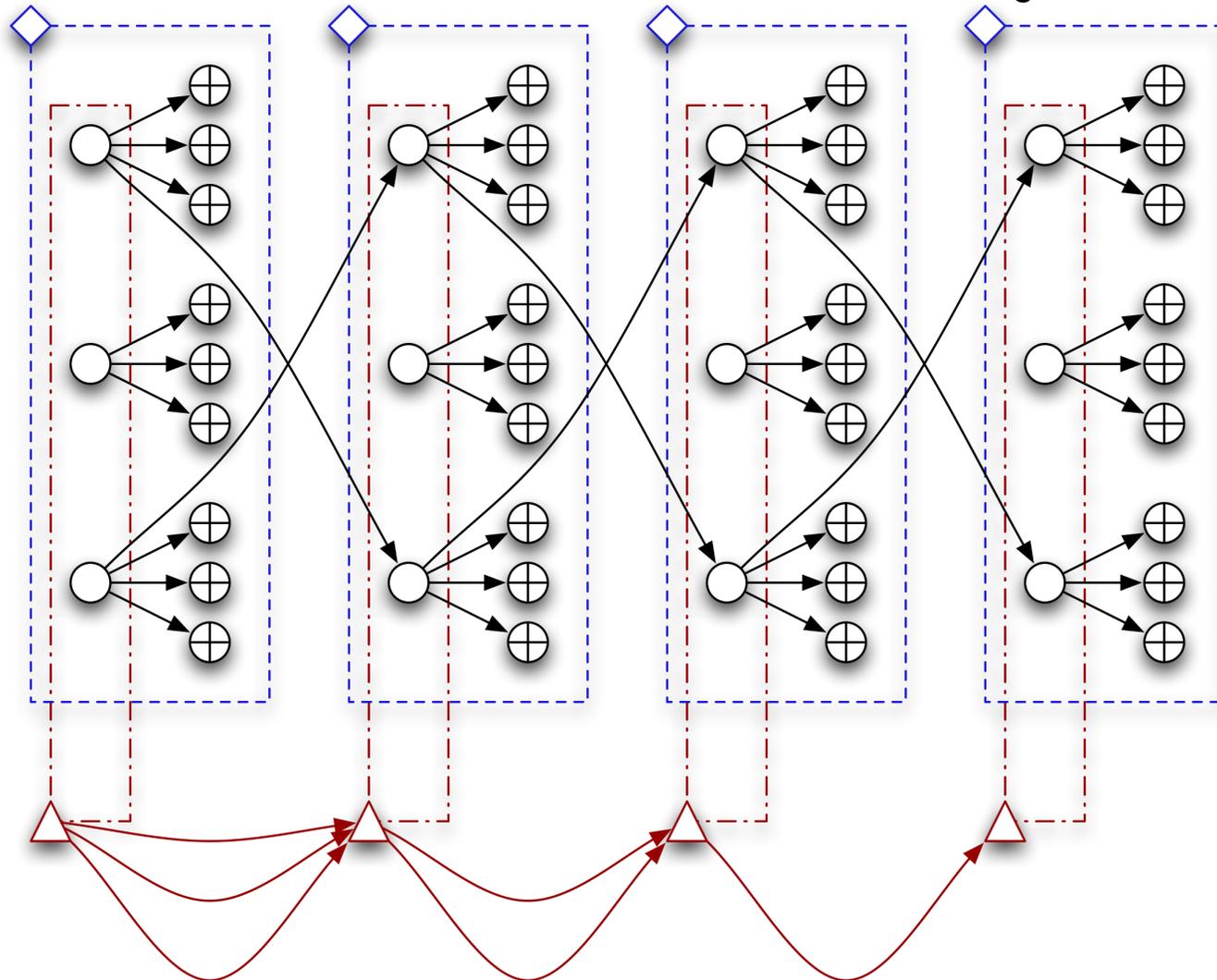


# Multi-period Wind



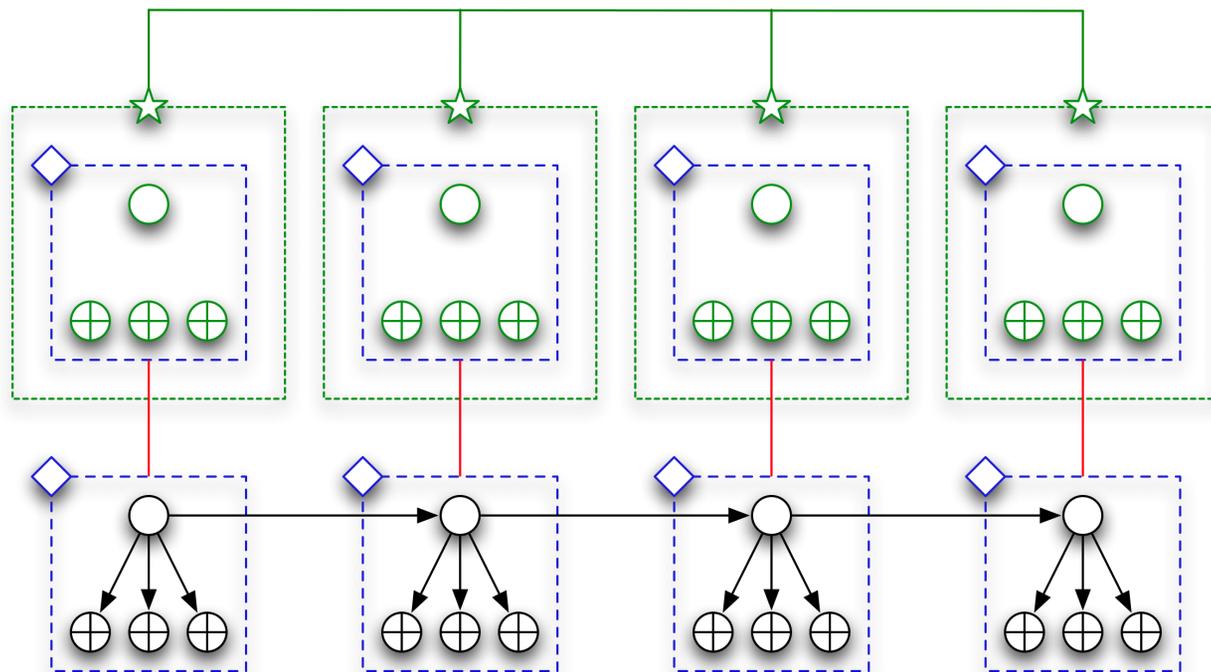
# With Storage / PHEV dispatch

 shared storage variable       conservation of energy and other storage constraints

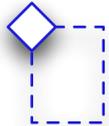


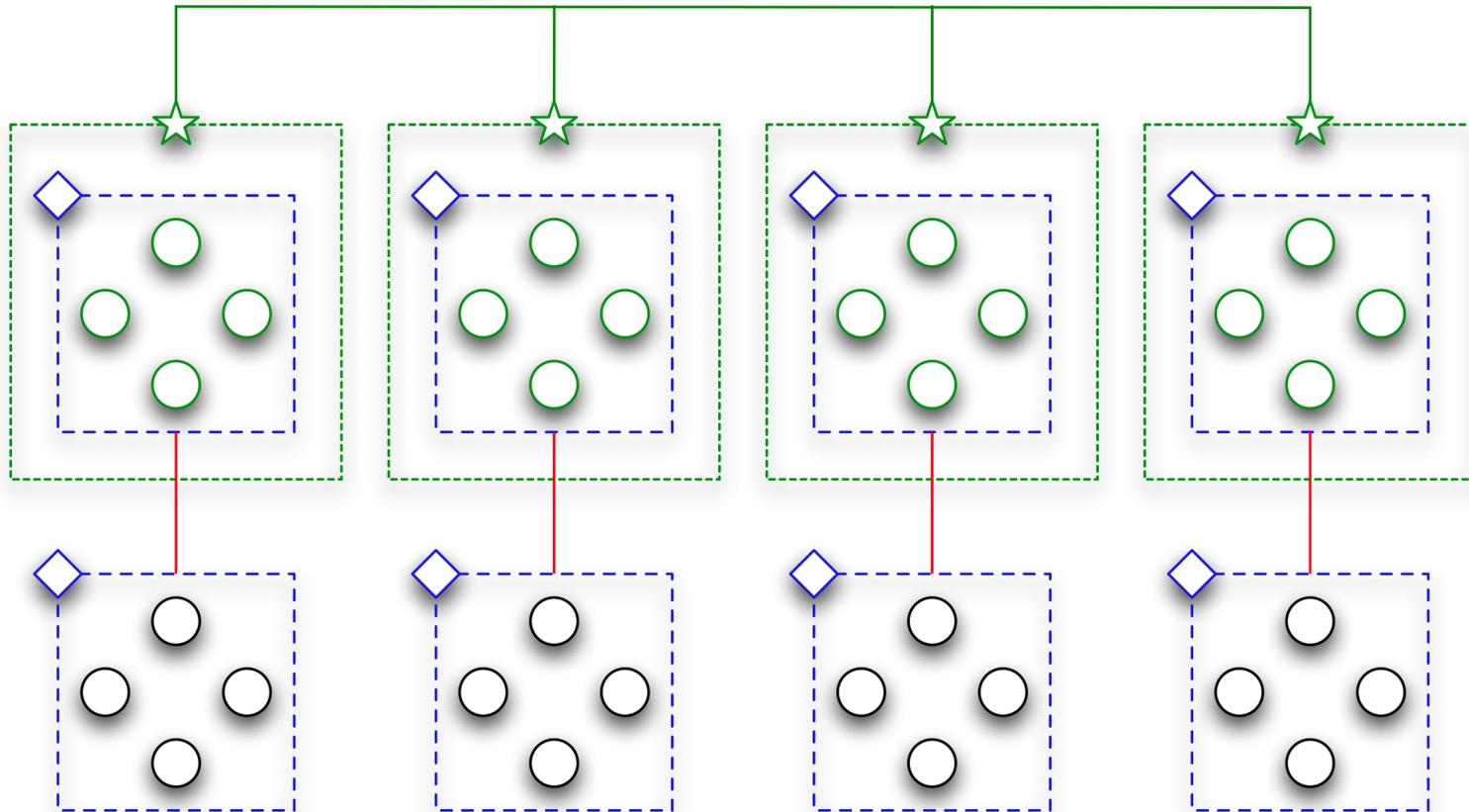
# Unit Commitment via Duplication + LR

-  integer variables & affected scenarios
-  constraints defining feasible set for integer variables
-  constraints relaxed by Lagrangian relaxation



# Generation Investment

 initial capacity variable + capacity to build



# Large Scale Problems

- Based on the Polish portion of the European UCTE interconnected network
  - 400, 220 and 110 kV networks
  - 2007-2008 winter evening peak
  - 3012 buses
  - 385 generators
  - 2271 loads
  - 3572 branches (201 transformers)

# Contingencies

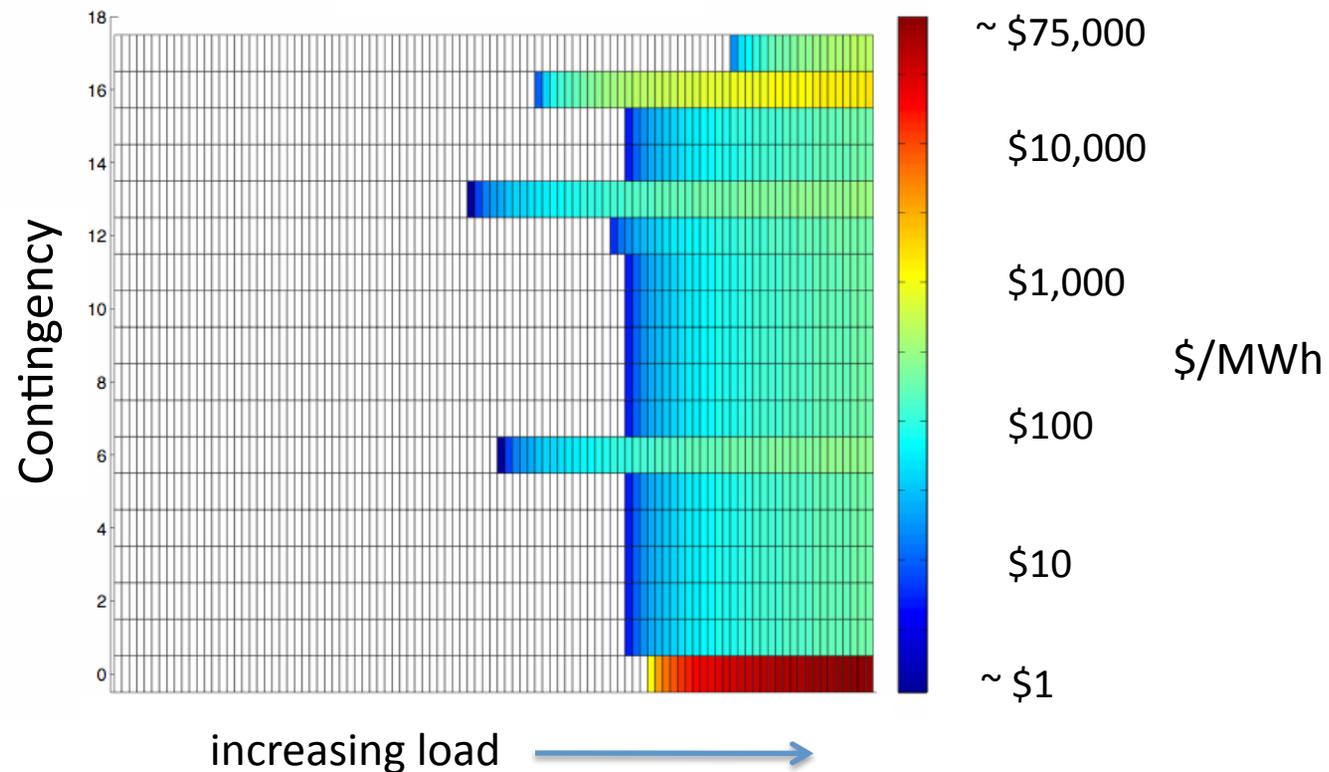
- Largest generator and line outages
  - 21 generator outages
  - 21 line outages
- Results in one huge network with 43 islands, each copies of original 3012 bus network with a different outage

# Equivalent OPF

- Equivalent network
  - 129,516 buses
  - 16,534 generators
  - 97,653 loads
  - 153,575 branches
- Optimization problem
  - 326,323 total optimization variables
  - 682,690 total constraints
    - 566,182 non-linear, 116,508 linear

# Expected Cost of Lost Load

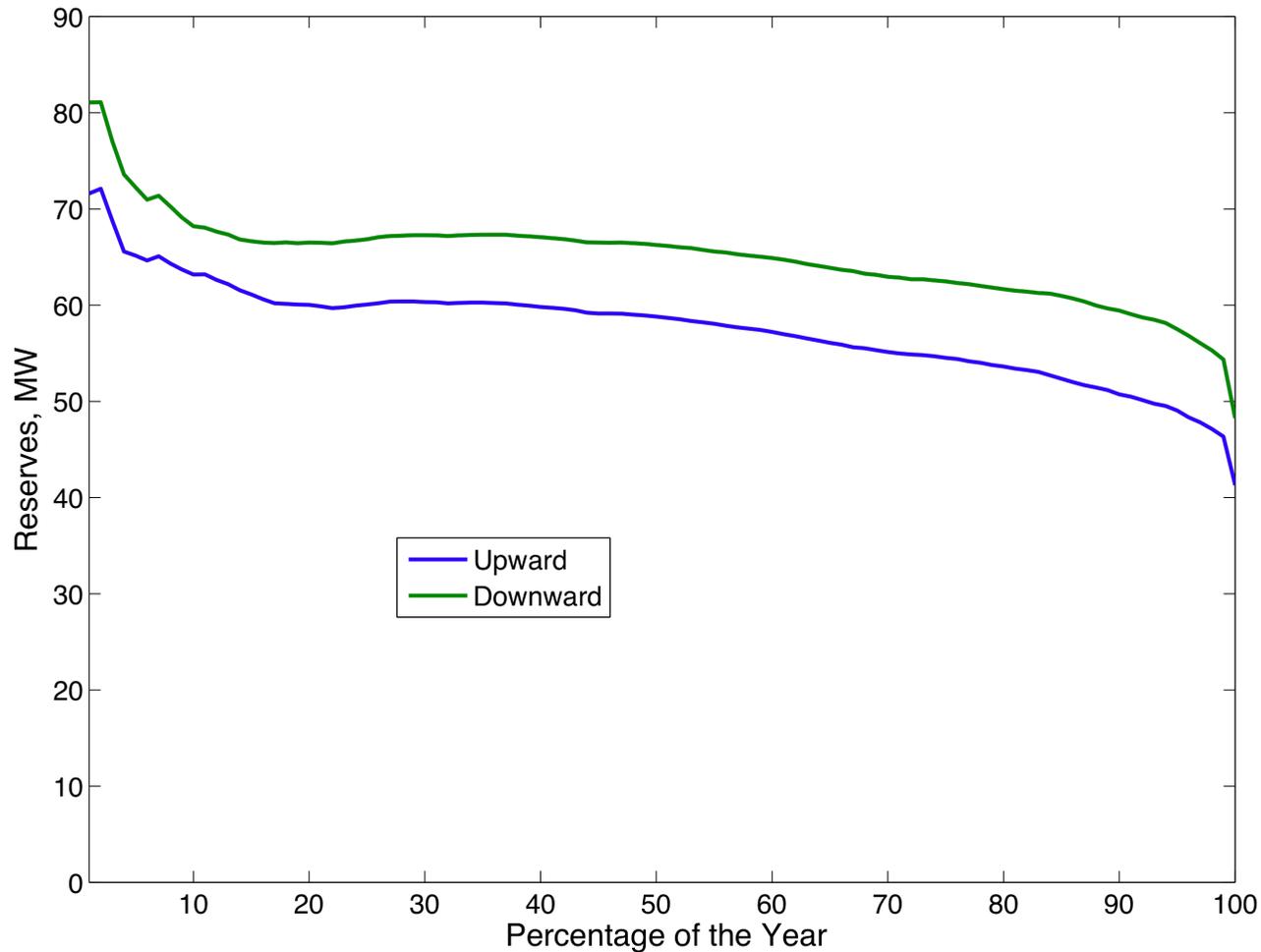
*weighted by probability of contingency*



- as load increases problems show up first in contingencies
- caused by a line limit

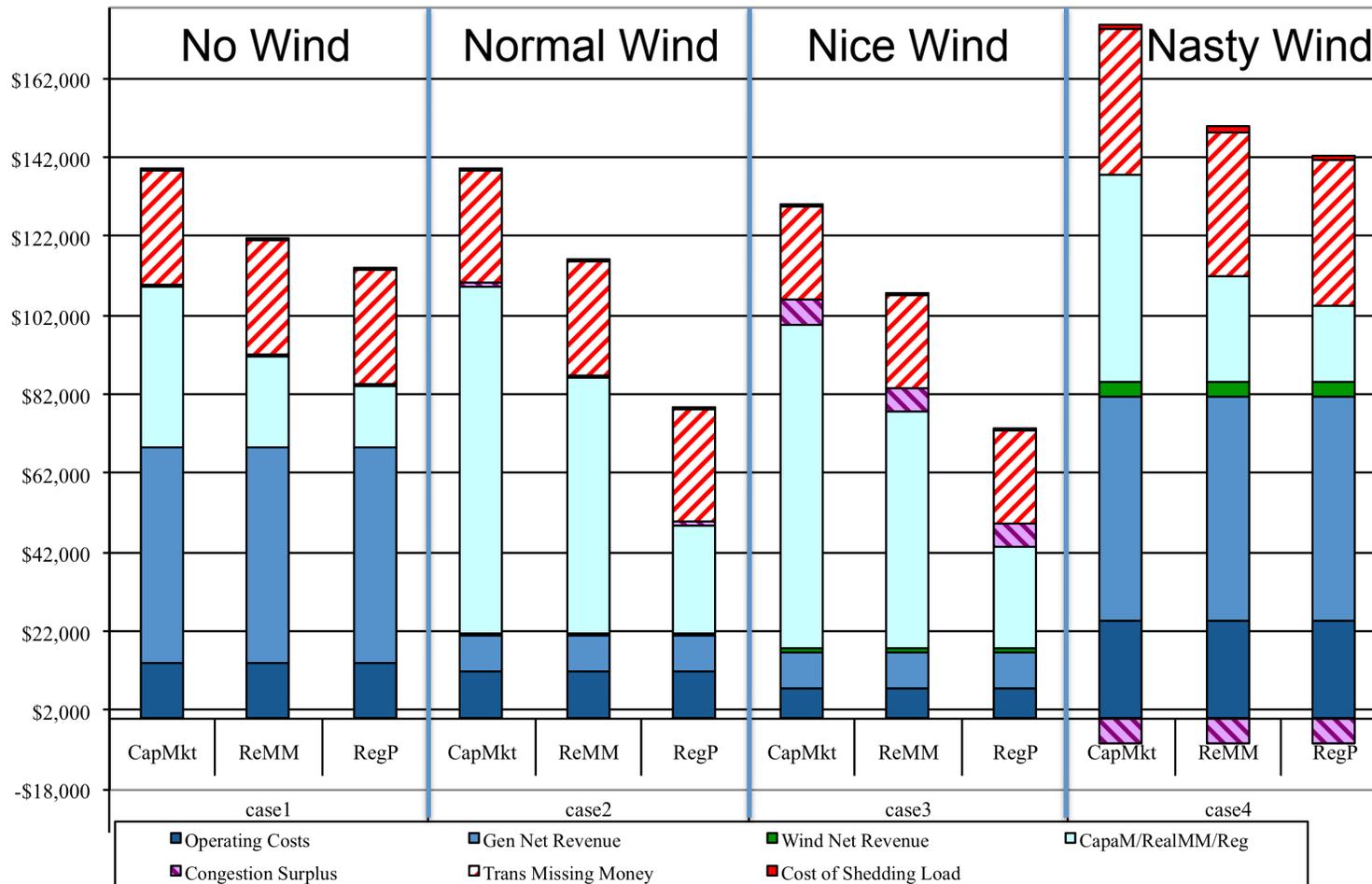
# Effect of Wind on Reserves

Expected Total Reserves  
(weighted by probability of wind forecast)  
lots of wind



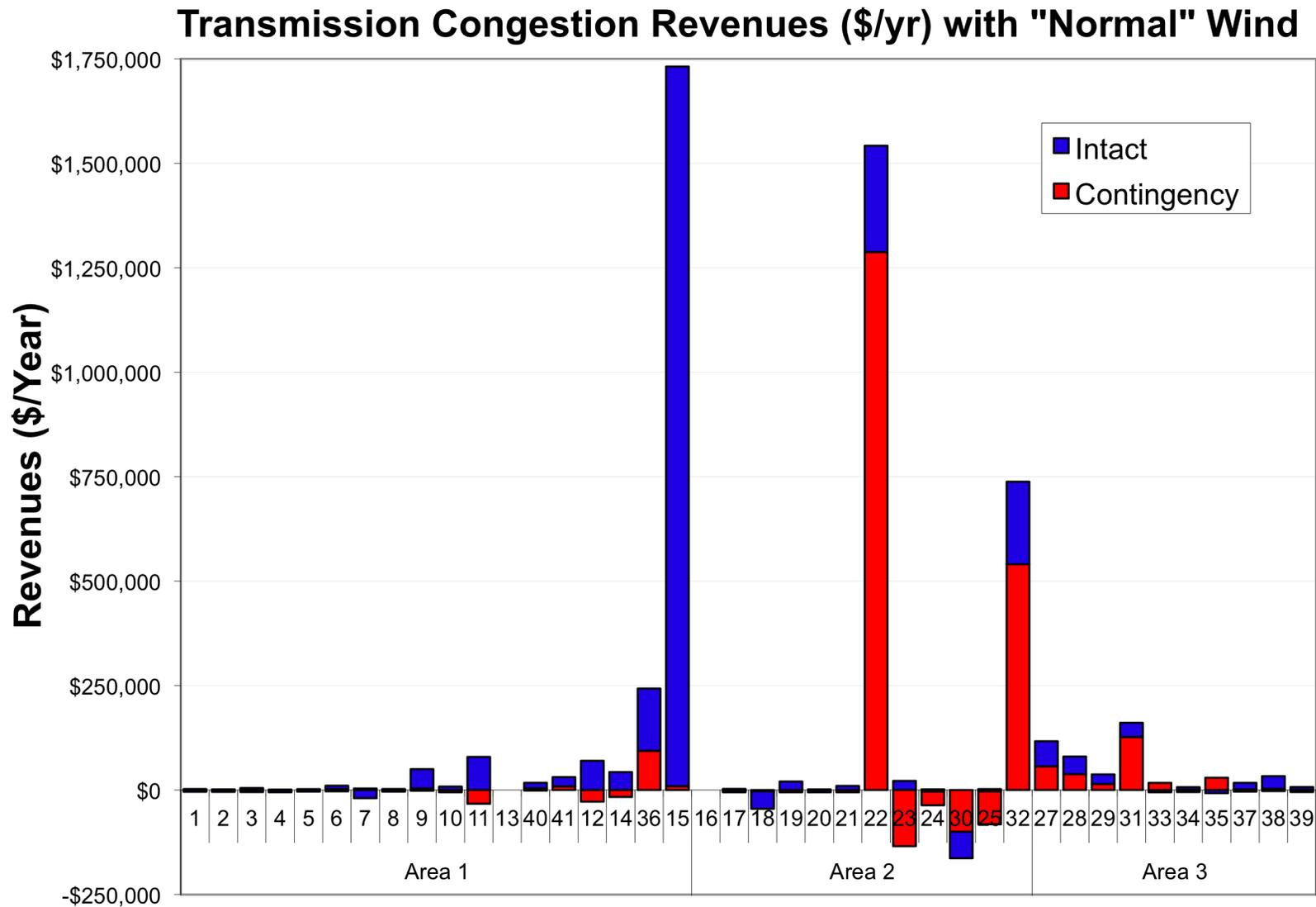
# Annual System Costs

Total Annual System Costs Paid by Customers (\$1000/Year)



# Expected Annual Congestion Revenues

*(based on line flows \* price differences)*



Questions?