Mixed Integer Programming:
Powering Unit Commitment Models

Edward Rothberg

Gurobi Optimization

Mixed Integer Programming

A mixed-integer program (MIP) is an optimization problem of the form:

Minimize	$c^T x$
Subject to	$Ax = b$
	$l \leq x \leq u$
	some x_j integer

Gurobi Optimization

Unit-Commitment Model

Electrical Power Industry, ERPI GS-6401, June 1989:

Mixed-integer programming (MIP) is a powerful modeling tool, "[MIP models] are, however, theoretically complicated and computationally cumbersome"

In other words: MIP looks nice, but it can't solve real problems

Unit-Commitment Model

- California Unit-Commitment Model
- 1999 Results (machine unknown):
  - 2-Day model: 8 hours, no progress
  - 7-Day model: 1 hour just to solve initial LP relaxation
- 2011 Results: Gurobi 4.5 ($700 desktop PC):
  - 7-Day model: proven optimal solution in 85 seconds
- Looks nice, and now can solve real problems

Gurobi Optimization

Model unitcal_7

```
Optimize a model with 48939 rows, 25755 columns and 127595 nonzeros
...
     0     0 1.9450e+07    0 1362          - 1.9450e+07     -      -    1s
     0     0 1.9536e+07    0 1027          - 1.9536e+07     -      -    3s
...
H    0     0                    3.118141e+07 1.9596e+07  37.2%     -   17s
H    0     0                    2.194163e+07 1.9596e+07  10.7%     -   18s
H    0     2                    1.998390e+07 1.9596e+07  1.94%     -   19s
     0     2 1.9596e+07    0  943 1.9984e+07 1.9596e+07  1.94%     -   19s
     3     5 1.9604e+07    2 1198 1.9984e+07 1.9596e+07  1.94%   771   20s
H   28    28                    1.991689e+07 1.9599e+07  1.60%   646   24s
    39    35 1.9612e+07   13  271 1.9917e+07 1.9599e+07  1.60%   579   25s
H   60    53                    1.989731e+07 1.9599e+07  1.50%   447   26s
H   61    55                    1.986798e+07 1.9599e+07  1.35%   440   26s
H   91    81                    1.974075e+07 1.9599e+07  0.72%   332   27s
H  117    38                    1.965010e+07 1.9605e+07  0.23%   266   29s
H  118    38                    1.964944e+07 1.9605e+07  0.22%   264   29s
H  119    38                    1.964934e+07 1.9605e+07  0.22%   262   29s
H  120    38                    1.964931e+07 1.9605e+07  0.22%   260   29s
*  121    38              43    1.964931e+07 1.9605e+07  0.22%   258   29s
   295    60 1.9621e+07    5  856 1.9649e+07 1.9607e+07  0.22%   160   31s
*  359    63              39    1.964468e+07 1.9607e+07  0.19%   149   31s
*  435   107              37    1.964403e+07 1.9607e+07  0.19%   136   32s
   534   186 1.9642e+07   10  479 1.9644e+07 1.9611e+07  0.17%   125   35s
H  547   129                    1.963642e+07 1.9611e+07  0.13%   124   35s
H  607   148                    1.963579e+07 1.9611e+07  0.13%   116   35s
H  764   235                    1.963557e+07 1.9611e+07  0.13%   100   37s
...
* 1327   412              45    1.963556e+07 1.9628e+07  0.04%   126   79s
  1432   429 1.9635e+07   25  453 1.9636e+07 1.9628e+07  0.04%   121   80s

Explored 2187 nodes (236902 simplex iterations) in 84.80 seconds
Optimal solution found (tolerance 1.00e-04)
```

Gurobi Optimization

MIP Keeps Improving

Gurobi Optimization

- Founded July, 2008
- Founders: Zonghao Gu, Ed Rothberg, Bob Bixby
- Product Releases:
  - Version 1.0: May 2009
    - Performance roughly equal to CPLEX 11.0
  - Version 2.0: October 2009
  - Version 3.0: April 2010
  - Version 4.0: November 2010
  - Version 4.5: April 2011

Gurobi Optimization

MIP Performance – Gurobi Internal Test Set

- Version to version improvements:
  (Geometric mean runtime over ~800 models in our internal model set that take more than 100s to solve)
  - Gurobi 1.0 –> 2.0: 2.2X
  - Gurobi 2.0 –> 3.0: 2.9X
  - Gurobi 3.0 –> 4.0: 1.3X
  - Gurobi 4.0 –> 4.5: 1.8X
- Continued improvement in our ability to solve difficult MIP models

Gurobi Optimization

MIP Performance – Public Benchmarks

- Gurobi 4.5 vs. CPLEX 12.2.0.2 and XPRESS 7.2 (>1.0 means Gurobi is faster)

	vs CPLEX 12.2.0.2			vs XPRESS 7.2		
	P=1	P=4	P=12	P=1	P=4	P=12
MIPLIB 2010	1.40X	–	1.20X	1.01X	–	1.15X
Feasibility	3.57X	–	–	8.16X	–	–
Infeasibility	–	–	2.62X	–	–	3.10X
Pathological*	–	–	0.90X	–	–	1.04X
MIQP	–	2.43X	–	–	2.22X	–

Gurobi Optimization

Implications of Improvements in MIP Technology For Power Industry Models

Gurobi Optimization

What’s Next?

- Consider an analogy – airlines
  - One of the earliest users of optimization
  - Consider…
    - Where they started
    - Where they are now
    - What we learned along the way

Where They Started

- Three high-ROI opportunities
  - Crew scheduling
  - Fleet assignment
  - Yield management (ticket prices)
- Much initial excitement about MIP (1970's)
- Followed soon after by great disappointment
  - Quite natural modeling paradigm
  - Little success in solving practical models
  - Sound familiar?

Simplification and Custom Strategies

Simplification / Custom Strategies

- Simplication
  - Leave out important details
- Custom strategies:
  - Lagrangian relaxation, LP-based, combinatorial, custom branching, specialized cutting planes, etc.
  - Tailor-made to crew/fleet/pricing problems
    - Built from a detailed understanding of the structure of the problem

Gurobi Optimization

Custom Strategies

- Pros (versus MIP model):
  - Can be quite effective
  - Attack problems that aren't tractable as MIPs
- Cons (versus MIP model):
  - Labor intensive
  - Typically quite brittle
    - Slight deviation from assumed problem structure often causes approach to fall apart
  - Often perceived as less neutral than a MIP model

Gurobi Optimization

Custom Approaches – Current Status

- As MIP technology improved…
  - Models routinely solved with no customization
  - Robust solutions in the presence of ‘side-constraints’
- Custom approaches no longer useful?
  - Quite the opposite:
    - Several general MIP procedures inspired by custom approaches
    - MIP solvers have assimilated the technology

Gurobi Optimization

Lessons Learned

- Mutual benefit from working together
  - Industry
    - Produced insights into how to solve specific models
    - Provided challenge sets
      - Representative, difficult datasets
  - MIP
    - Generalized strategies to work across a broad spectrum of MIP models
- Result: robust solutions to formerly impossible models

Gurobi Optimization

Datasets?

- New MIP benchmark set (MIPLIB2010)
  - Benchmark set – has a big influence of MIP algorithm development
  - Gurobi pointed out that early proposed set contained no power industry models
    - An important problem class that should be represented
  - Call went out for publicly available models
  - Result: unitcal_7 (1999)

Growing the Models

Model Growth

- Once the original problems were tractable, new MIP capabilities used to expand the model
  - Larger: more detailed representation of the problem
  - Broader: integrate multiple parts of the organization into the model
  - Faster: move from planning to real-time optimization
  - More general objectives: model non-linear objectives and constraints using piecewise-linear functions

Gurobi Optimization

Future Challenges

Current and Future Challenges

- Better cooperation between optimization models
  - Integration has its limits
- Dealing with uncertainty (weather, mechanical problems, etc.)
  - Techniques exist (stochastic programming, robust optimization)
  - Limited practical success so far

Gurobi Optimization

Moving Forward

Questions

- Insights from custom heuristics?
  - Any that might be integrated into general MIP?
- Challenge sets?
  - Can you share datasets that capture your next challenge?
- Model growth?
  - How are power industry models likely to change in the future?

Gurobi Optimization