Economic Interpretation of Demand Curves in Multi-product Electricity Markets

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Demand Curves in Electricity Markets

- Reliability products such as reserves and capacity in electricity markets lack demand-side bids due to the public-good nature of reliability
- ISOs/RTOs often model the demand of a reliability product by a fixed requirement (with constraint-violation penalties), or the administratively-set demand curve
- Demand curves are gaining traction as ISO/RTO markets are moving away from fixed requirements

Fixed Requirement vs. Demand Curve

Fixed Requirement with Penalty	Demand Curve
One or several penalty **values**	A price-quantity **function**
Constraint-based	**Product-**based
Mainly for **Feasibility** (high penalty for constraint violation)	Mainly for **Pricing** (no concept of violation)

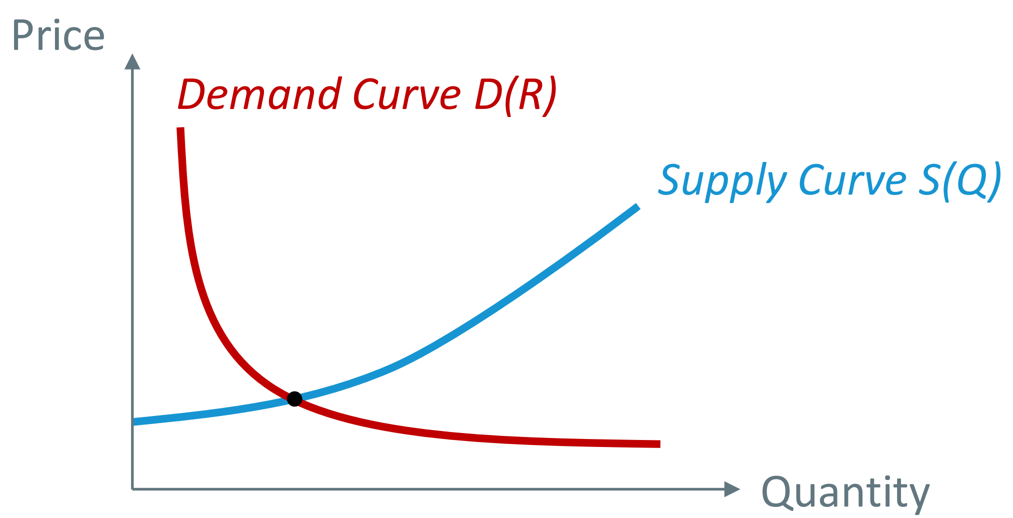
- It is tempting to view demand curves as a simple upgrade of penalty *values* to more sophisticated penalty *functions*, ignoring the fundamental shift from the **constraint-based violation costs** to the **product-based economic benefits**

<u>Proper economic interpretation of demand curves is critical!</u>

Demand Curve Interpretation Of a Single Product

- For a market with a single reliability product or the product is independent of others, the monotonically decreasing demand curve represents the product's **marginal reliability benefit**

The market clearing objective: $$maximize \int_0^R D(R)dR - \int_0^Q S(Q)dQ$$



Demand Curve Interpretation of Coupled Products

- A reserve or capacity market usually involves multiple coupled products, e.g., spinning and non-spinning reserves, local and system capacities
- The typical coupling relation is "**substitution**", i.e., a high-quality product *A* (e.g., spinning reserve or local capacity) can be used to substitute a low-quality product *B* (e.g., non-spinning reserve or system capacity)
- The interpretation of demand curves associated with coupled products has not received much attention, as discussions have been focused on the choice of demand curve parameters

A Stylized Two-Product Market Model

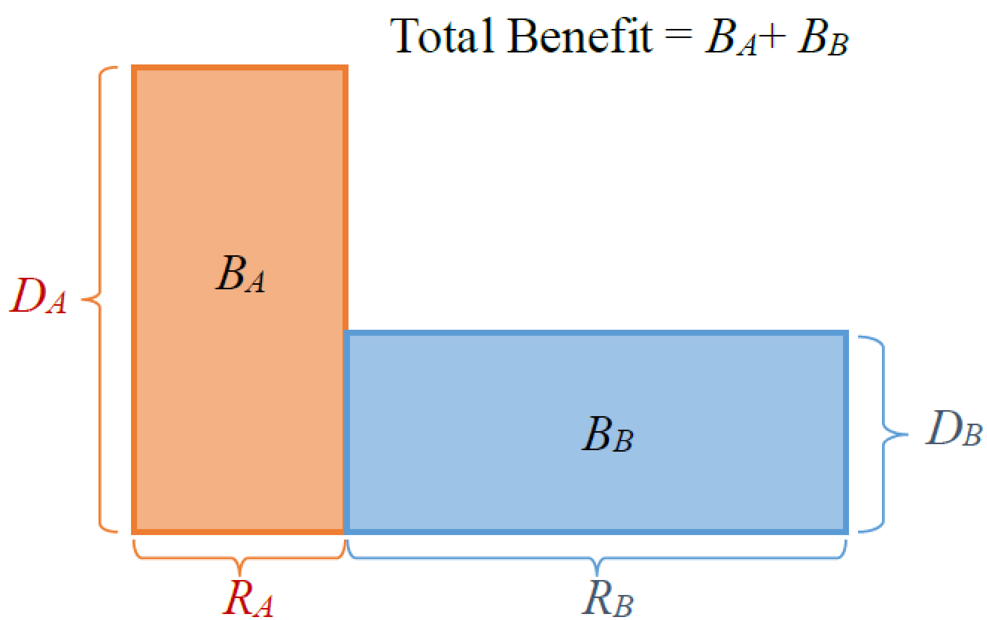
- Consider a two-product market with the high-quality product *A* and the low-quality product *B*
- The supply quantities of the two products are Q_A and Q_B, while the demand quantities of the two are R_A and R_B
- The substitution of product *A* for product *B* can be represented by:

$$Q_A \geq R_A$$

$$Q_A + Q_B \geq R_A + R_B$$

Demand Curve as The Marginal Benefit of Individual Product – Interpretation I

- An extension of the single-product demand curve interpretation: **Product *A*'s demand curve $D_A(R_A)$ represents *A*'s marginal reliability benefit,** and **Product *B*'s demand curve $D_B(R_B)$ represents *B*'s marginal reliability benefit**



The total benefit of the two products is the sum of individual product benefits

Market Clearing Under Demand Curve Interpretation I

$$\min_{Q_A,Q_B,R_A,R_B} \underbrace{C_A(Q_A) + C_B(Q_B)}_{\text{Total Cost}} - \underbrace{\left[\int_0^{R_A} D_A(R)dR + \int_0^{R_B} D_B(R)dR\right]}_{\text{Total Benefit (TB)}}$$

$$s.t. \qquad Q_A \geq R_A \qquad (\lambda_A)$$

$$Q_A + Q_B \geq R_A + R_B \qquad (\lambda_B)$$

$$Q_A \in \Omega_A,\ Q_B \in \Omega_B$$

$$R_A, R_B \geq 0.$$

λ_A and λ_B are multipliers associated with corresponding constraints

Market Clearing Prices Under Interpretation I

- Market Clearing Price (MCP) for each product is defined as the derivative of the optimal objective cost with respect to the perturbation of corresponding demand, i.e.,

$$MCP_A \equiv \lambda_A + \lambda_B.$$

$$MCP_B \equiv \lambda_B.$$

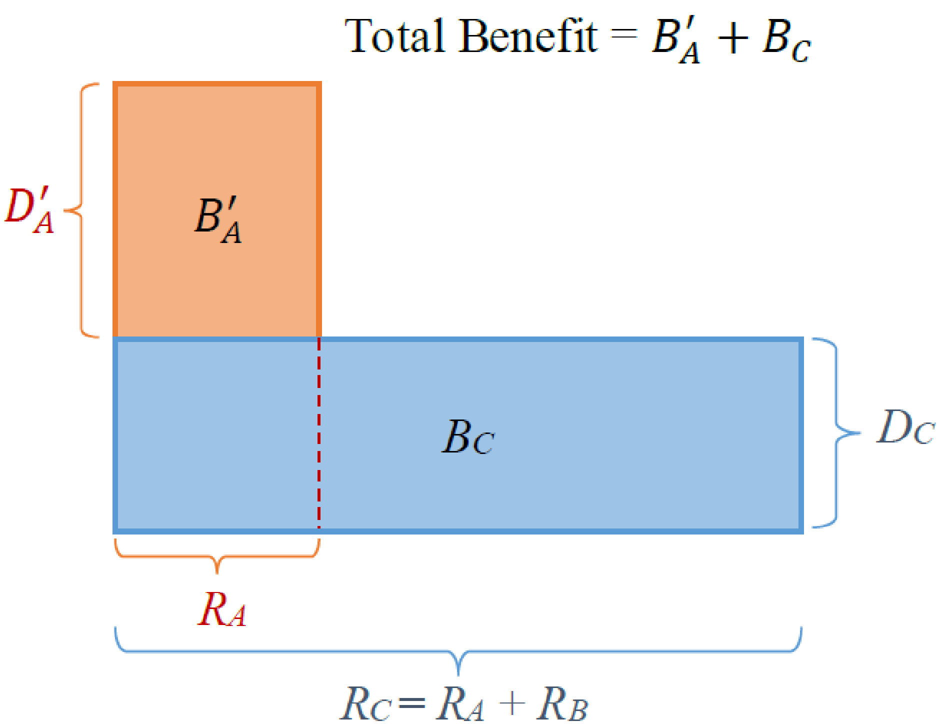
- Assuming problem convexity, the optimality conditions of the clearing problem yield $D_A(R_A^{opt}) = \lambda_A + \lambda_B, \quad D_B(R_B^{opt}) = \lambda_B.$
- Therefore,

$$\boxed{\begin{aligned} D_A(R_A^{opt}) &= MCP_A, \\ D_B(R_B^{opt}) &= MCP_B. \end{aligned}}$$

The demand curve interpretation is consistent with the market clearing price

Demand Curve as Additional Benefit of One Product Over Another – Interpretation II

- The Total Benefit (TB) can also be formed as the basic benefit of both products providing low-quality service, plus the additional benefit of the high-quality product *A*



Demand curve D_C represents the "basic" marginal benefit of both products

Demand curve D'_A represents the "additional" marginal benefit of the high-quality product *A* over the basic marginal benefit.

Market Clearing Under Demand Curve Interpretation II

$$\min_{Q_A,Q_B,R_A,R_C} \underbrace{C_A(Q_A) + C_B(Q_B)}_{\text{Total Cost}} - [\int_0^{R_A} \underbrace{D'_A(R)dR + \int_0^{R_C} D_C(R)dR]}_{\text{Total Benefit (TB)}}$$

$$s.t. \quad Q_A \geq R_A \quad (\lambda_A{}')$$

$$Q_A + Q_B \geq R_C \quad (\lambda_C)$$

$$Q_A \in \Omega_A,\ Q_B \in \Omega_B$$

$$R_A, R_C \geq 0.$$

- $\lambda_A{}'$ and λ_C are multipliers associated with corresponding constraints

Market Clearing Prices Under Interpretation II

- Market Clearing Prices (MCP) for the products are defined as

$$MCP_A \equiv \lambda'_A + \lambda_C,$$

$$MCP_B \equiv \lambda_C.$$

- Assuming problem convexity, the optimality conditions of the clearing problem yield $D'_A\left(R_A^{opt}\right) = \lambda'_A$, $D_C\left(R_C^{opt}\right) = \lambda_C$.
- Therefore,

$$\boxed{\begin{aligned} D'_A\left(R_A^{opt}\right) &= MCP_A - MCP_B, \\ D_C\left(R_C^{opt}\right) &= MCP_B. \end{aligned}}$$

The demand curve interpretation is consistent with the market clearing price

Summary of Two Demand Curve Interpretations

- The two **different compositions** of the Total Benefit (TB) lead to demand curves with different interpretations
  - TB as the sum of individual product benefits (Interpretation I)
  - TB as the sum of basic product benefit and the premium benefit of the high-quality product (Interpretation II)
- Each interpretation is also associated with a corresponding market clearing formulation
- The two interpretations are generally **inequivalent**

Comparison of Demand Curve Interpretations

- The two total benefit compositions are equivalent *if and only if* the basic benefit demand curve function ***Dc* is constant**

$$\underbrace{\int_0^{R_A} D_A(R)dR + \int_0^{R_B} D_B(R)dR}_{\text{Composition I}} = \underbrace{\int_0^{R_A} D_A'(R)dR + \int_0^{R_A+R_B} D_C(R)dR}_{\text{Composition II}}, \quad \forall (R_A, R_B)$$

$$\Leftrightarrow D_c \text{ is constant}$$

- One demand curve representation can be equivalently translated into another if and only if *Dc* is constant:

$$D_A'(R_A) = D_A(R_A) - D_C$$

Choice of Demand Curve Interpretation

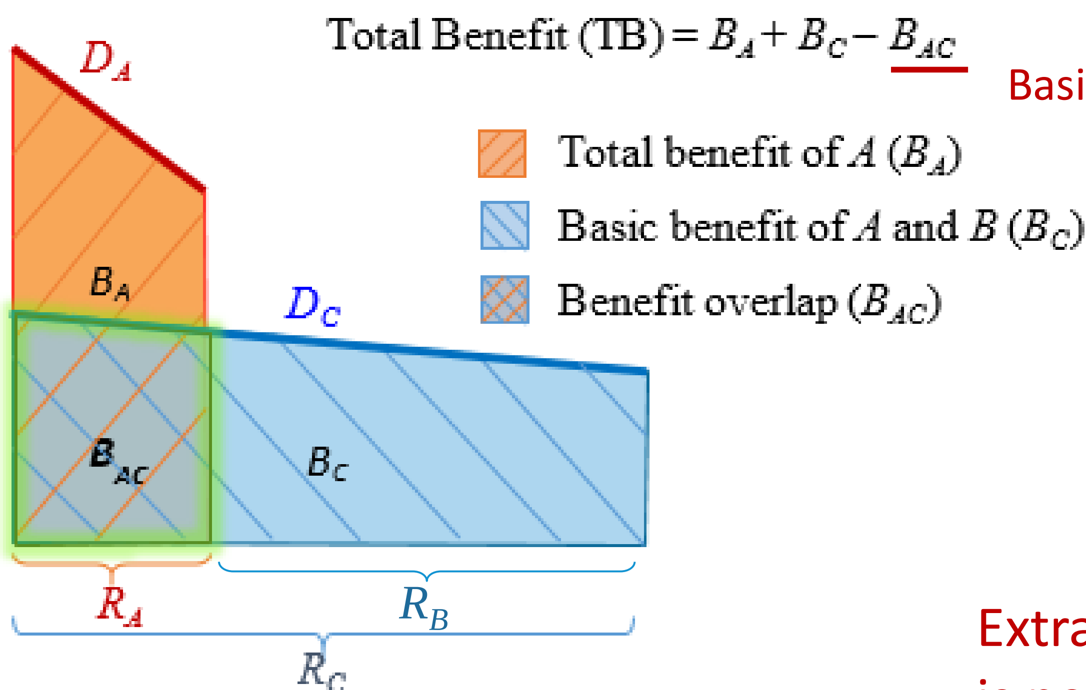
- Under both demand curve interpretations, the total benefit is considered to be additively separable in terms of R_A and R_B under Interpretation I, and R_A and R_C under Interpretation II
- In reality, the multi-variate total benefit function may not be separable
- The choice of demand curve interpretation could depend on whether the total benefit composition under the interpretation is a good approximation of the actual benefit function, and the complexity of constructing and clearing the demand curves under a particular interpretation

Practical Implications

- The construction of demand curves and the use of them in market clearing must be **consistent** with their economic interpretations
- In practice, the demand curves simply evolved from fixed requirements without recognizing their economic meaning may not fit exactly into either of the two interpretations
  - A typical situation under fixed requirements has a requirement R_A for the high-quality product *A,* and a total requirement R_c for both products *A* and *B*
  - In transition to the demand curves, the market is presented with a demand curve $D_A(R_A)$ for high-quality product *A,* and a demand curve $D_C(R_C)$ for the total basic benefit of both products

The Form of Total Benefit (TB)

- The two benefits (integrals of the two demand curve functions D_A and D_C) has an overlapped area B_{AC}



Extra assumption on B_{AC} is needed to form *TB*

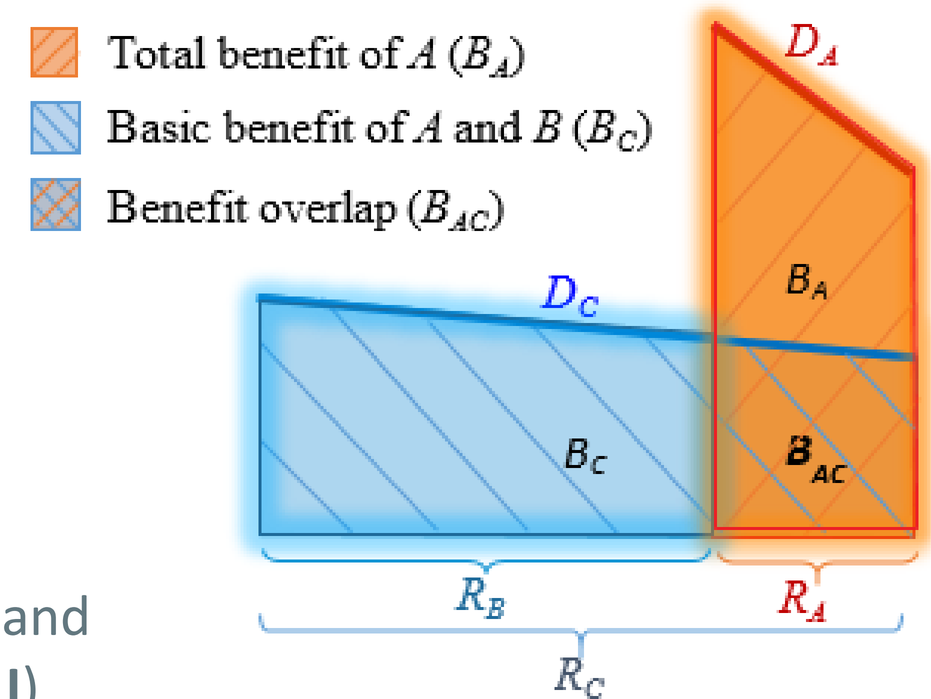
$$TB=\int_0^{R_A} D_A(R)dR + \int_0^{R_C} D_C(R)dR - \boxed{B_{AC}}$$

Assumption I on B_{AC}

- B_{AC} (*benefit of B, i.e.,* B_C*-*$B_{AC} = \int_0^{R_B} D_C(R)dR$) is associated with the **low** (*high*) part of the total basic benefit function B_C

$$B_{AC} = \int_{R_C - R_A}^{R_C} D_C(R)dR$$

Total Benefit (TB) = $B_A + B_C - B_{AC}$



Benefit of *A* Benefit of *B*

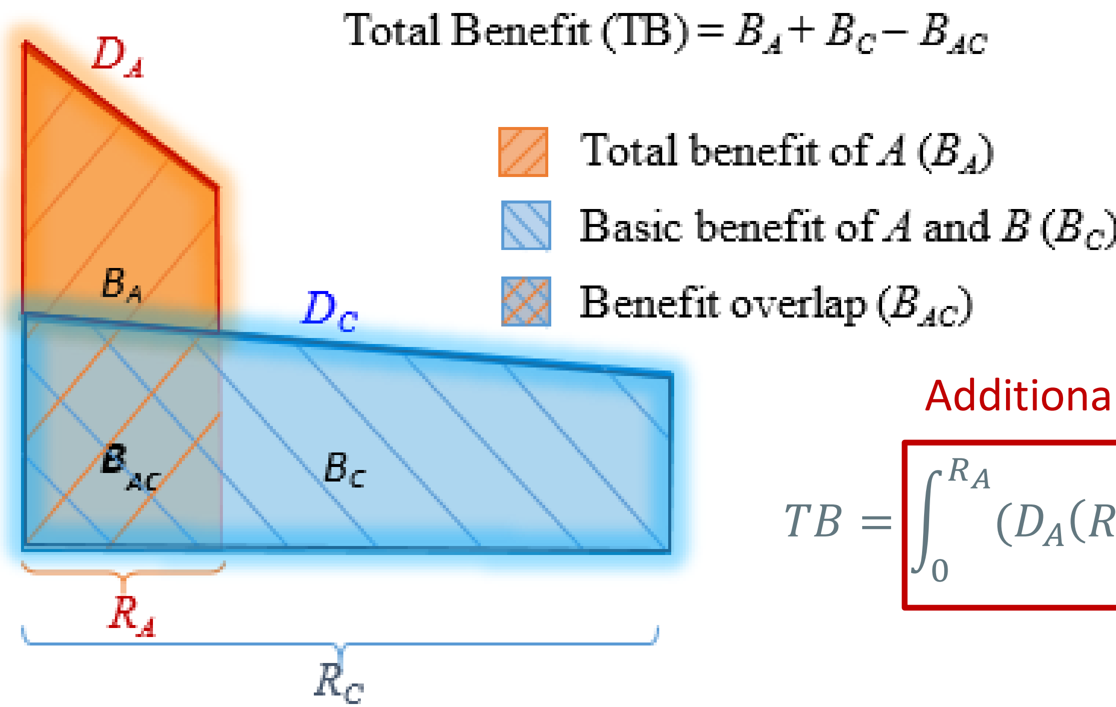
$$TB = \boxed{\int_0^{R_A} D_A(R)dR} + \boxed{\int_0^{R_B} D_C(R)dR}$$

D_A and D_C, respectively, represent *A* and *B*'s marginal benefits (**Interpretation I**)

Assumption II on B_{AC}

- B_{AC} (*benefit of B, i.e.,* B_C*-*B_{AC}) is associated with the **high** (*low*) part of the total basic benefit function B_C

$$B_{AC} = \int_0^{R_A} D_C(R)\,dR$$



Additional benefit of *A*

Basic benefit of *A* and *B*

$$TB = \underbrace{\boxed{\int_0^{R_A} (D_A(R) - D_C(R))\,dR}}_{\text{Additional benefit of } A} + \underbrace{\boxed{\int_0^{R_C} D_C(R)\,dR}}_{\text{Basic benefit of } A \text{ and } B}$$

$D'_A \equiv D_A - D_C$ represents *A*'s additional marginal benefit, and D_C represents the total basic marginal benefit (**Interpretation II**)

Compare the Assumptions on B_{AC}

$$TB = \int_0^{R_A} D_A(R)\,dR + \int_0^{R_C} D_C(R)\,dR - \boxed{B_{AC}}$$

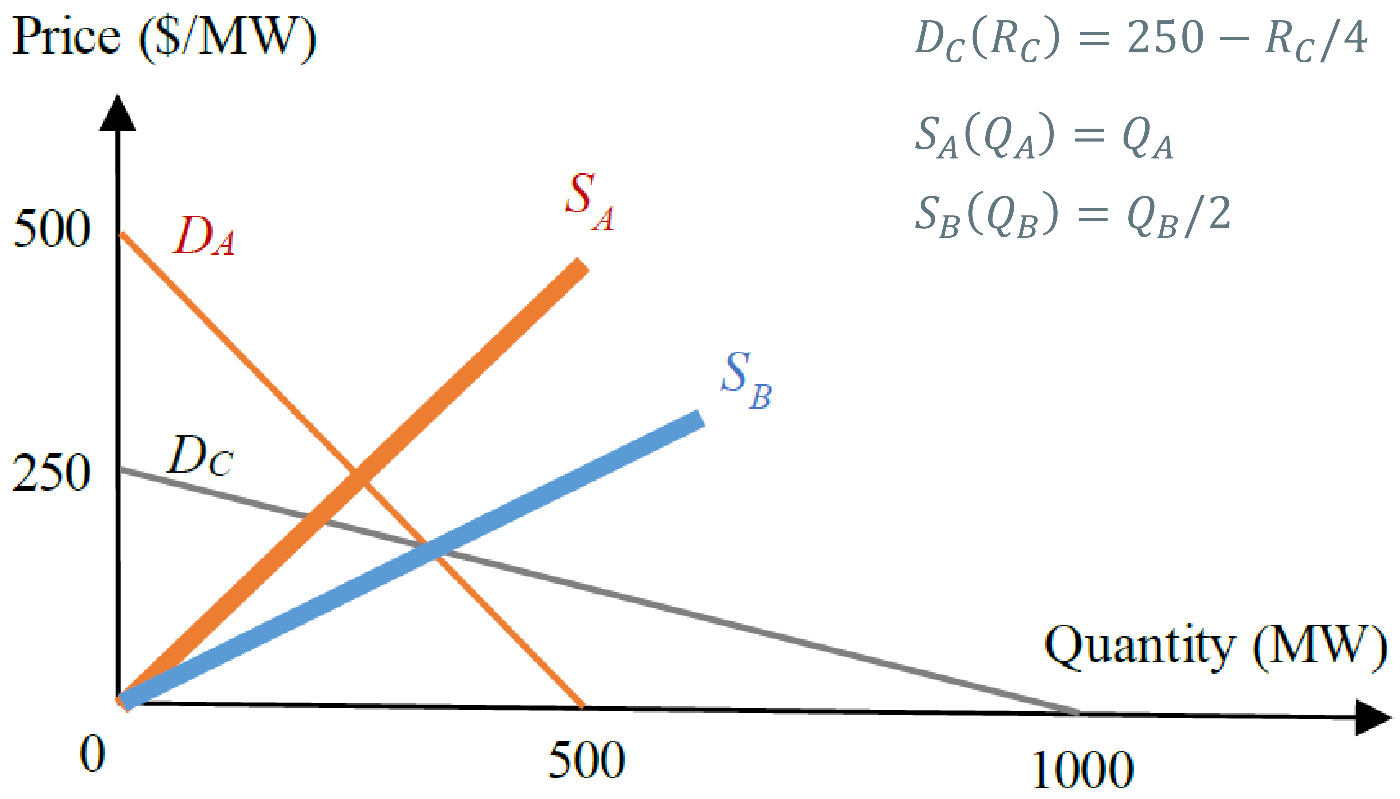
Basic benefit of *A*

Assumption I	Assumption II
B_{AC} is associated with the **low** part of the total basic benefit function B_C	B_{AC} is associated with the **high** part of the total basic benefit function B_C
$B_{AC} = \int_{R_C - R_A}^{R_C} D_C(R)dR$	$B_{AC} = \int_0^{R_A} D_C(R)dR$
$TB = \int_0^{R_A} D_A(R)dR + \int_0^{R_B} D_C(R)dR$	$TB = \int_0^{R_A} (D_A(R) - D_C(R))dR + \int_0^{R_C} D_C(R)dR$
D_A and D_C, respectively, represent *A* and *B*'s marginal benefits (**Interpretation I**)	$D_A' \equiv (D_A - D_C)$ represents A's additional marginal benefit, and D_C represents the total basic marginal benefit (**Interpretation II**)

The two assumptions converge when D_c is constant

An Illustrative Example

- Consider a two-product market with demand and supply curves illustrated below



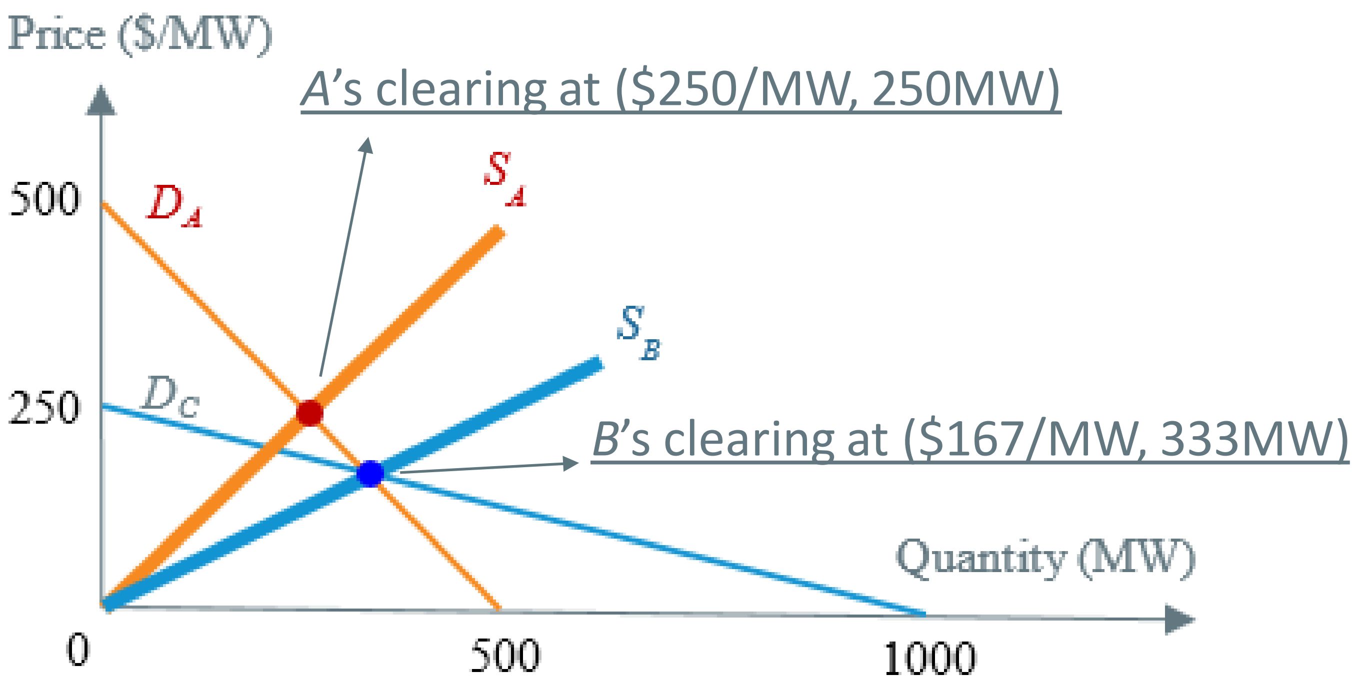
$$D_A(R_A) = 500 - R_A$$

$$D_C(R_C) = 250 - R_C/4$$

$$S_A(Q_A) = Q_A$$

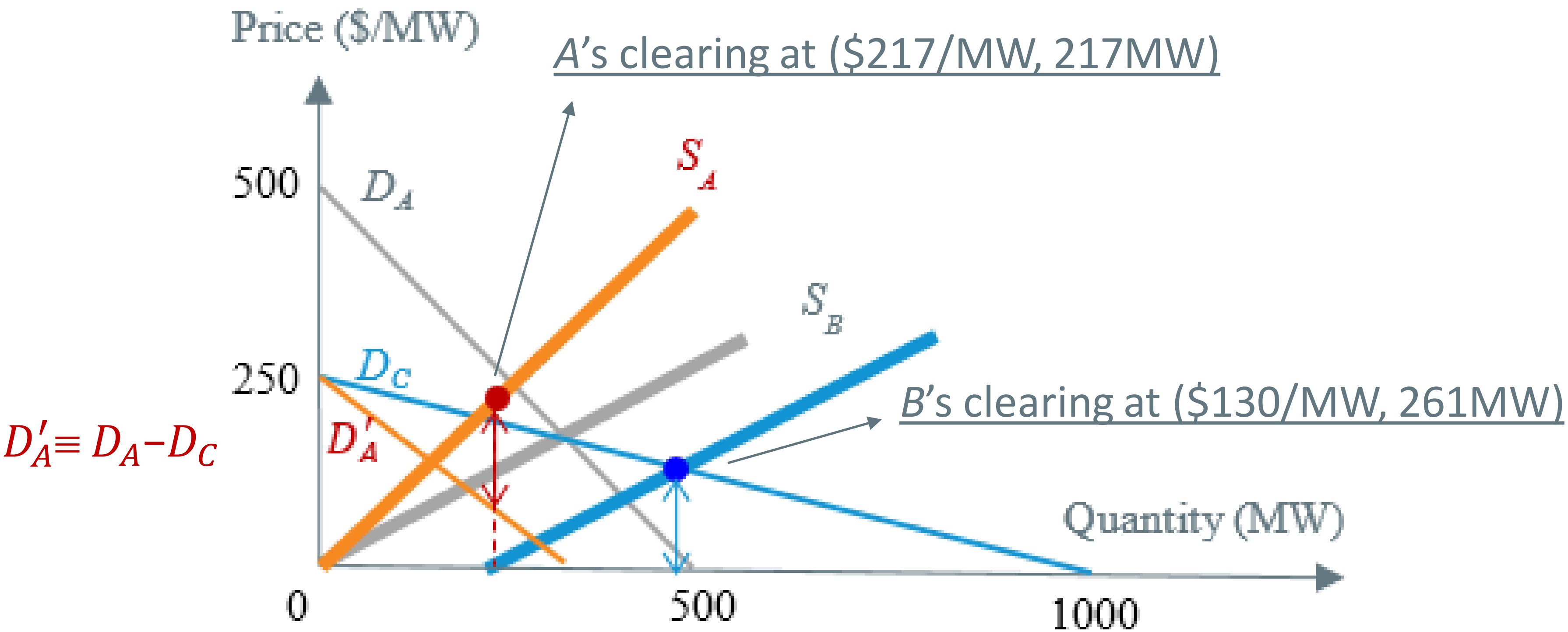
$$S_B(Q_B) = Q_B/2$$

Clearing Under Assumption/Interpretation I



- High-quality product *A* clears at the intersection of S_A and D_A, with a higher intersection price than that of the intersection of S_B and D_C (Low-quality product *B*'s clearing point)

Clearing Under Assumption/Interpretation II



- The clearing occurs $S_A(R_A) - D'_A(R_A) = S_B(R_B) = D_C(R_A + R_B)$, i.e., by increasing R_A the marginal surplus reduction from S_A and D'_A equals the marginal surplus gain from S_B and D_C

Comparison of Results under Two Assumptions

Assumption/Interpretation I	Assumption/Interpretation II
Demand benefit of B associated with high-part of total basic demand curve D_C	Demand benefit of B associated with low-part of total basic demand curve D_C
$Q_A^* = R_A^* = 250MW, MCP_A = \$250/MW$	$Q_A^* = R_A^* = 217.4MW, MCP_A = \$217.4/MW$
$Q_B^* = R_B^* = 333.3MW, MCP_B = \$166.7/MW$	$Q_B^* = R_B^* = 260.9MW, MCP_B = \$130.4/MW$
Total Surplus = $104,166.7	Total Surplus = $86,956.5

- Assumption-I with a higher valuation of product B leads to more cleared quantity of the product
- The total surplus under Assumption-I is higher due to less counting of the overlapped basic benefit of product A
- Different demand curve assumptions/interpretations lead to drastically different clearing results – **Important to have clear interpretations**!

Conclusion

- In the transition from constraint-based penalty factors to product-based demand curves, a proper interpretation of demand curves is fundamental to forming the demand benefit and social surplus, especially with multiple coupled products
- Using a two-product market, we reveal two different interpretations of the demand curves, each associated with a specific form of the market clearing, implying that the construction of demand curves and the use of them in market clearing must be consistent with their interpretation
- The practical impact of this work is demonstrated by analyzing a market with a demand curve for the high-quality product and a demand curve for the total basic benefit of both products

For Further Reading

- F. Zhao, T. Zheng, E. Litvinov, “Constructing Demand Curves in Forward Capacity Market,” IEEE Transactions on Power Systems, March 2017, Vol.33, No. 1, pp. 525-535.
- F. Zhao, “Real-time Reserve Demand Curves (RDC),” FERC Technical Conference, Jun. 25-27, 2019
- F. Zhao, T. Zheng, E. Litvinov, “Economic Interpretation of Demand Curves in Multi-product Electricity Markets,” (to appear on *IEEE Transactions on Power Systems*) [Available] http://www.optimization-online.org/DB_FILE/2020/02/7613.pdf