Transient Optimization of Large Natural Gas Pipeline Networks using Linear Programming

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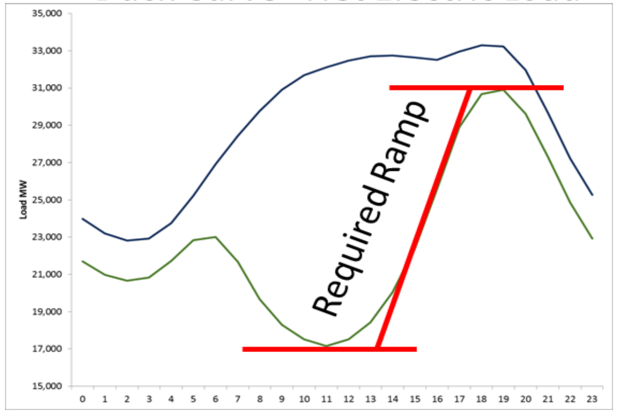
Outline

- Background and motivation
- Generic pipeline optimization model
- Linearization of flow-pressure relationships
- Linearization of compressor models
- Accuracy and computational performance
- What's next?

Increasing Dependency of Electric Industry on Natural Gas as a Fuel

- Factors:
  - Natural gas in the US continues to be available and affordable
  - On-going replacement of retiring coal and nuclear plants with a mix of renewable and conventional gas-fired generation
  - Growing volumes of gas use
  - Growing variability of gas use due to load following and other A/S services provided by gas-fired power plants
  - Need to procure gas on-demand close to, or in, real-time
  - Resource adequacy, reliability and resilience of electrical systems depends on resource adequacy, reliability and resilience of serving them pipeline networks

"Duck Curve" Net Electric Load



- Required ramp must be supported by gas pipeline network serving ramping generators
  - Need to be able to manage intra-day and even sub-hourly transient processes within pipeline networks

We need a variety of fast and scalable computational methods for modeling, optimization, control and market support over large regional and even continental size pipeline networks

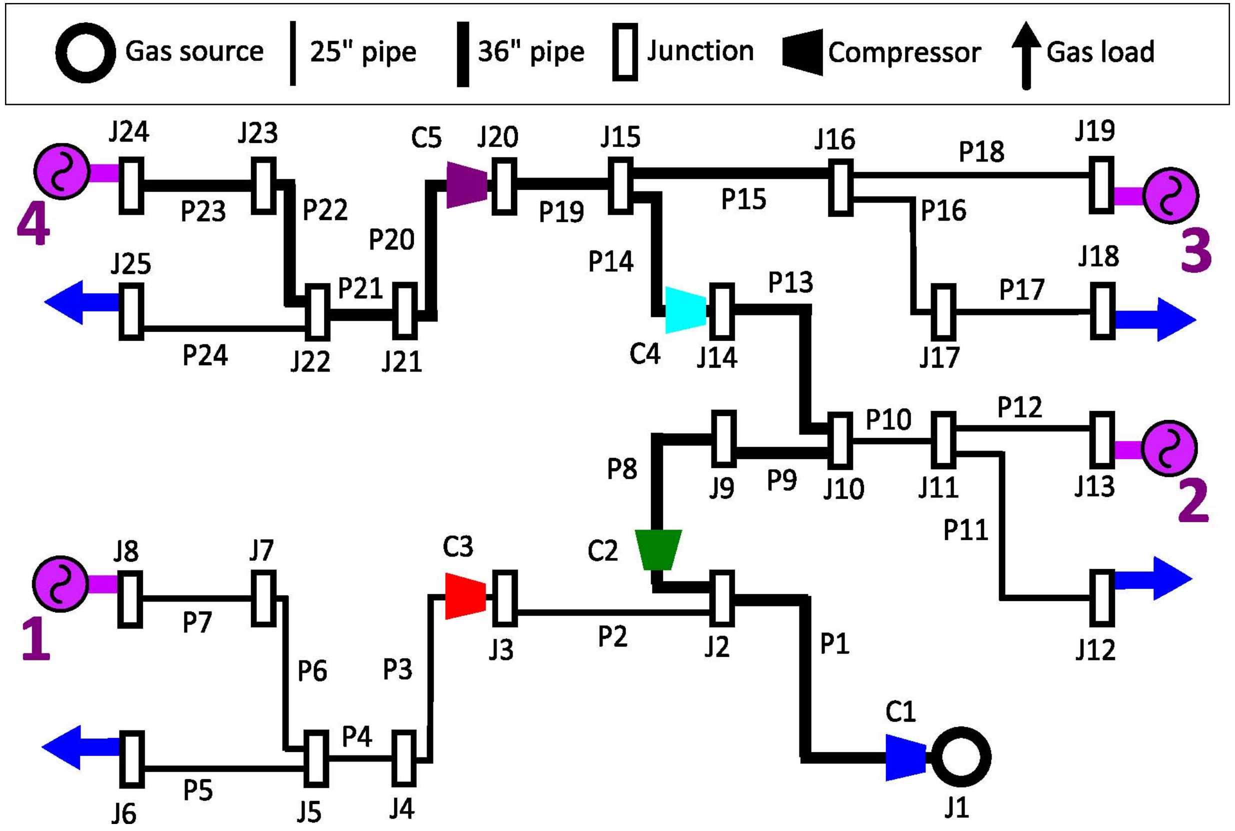
Available pipeline modeling tools

- **Physical flow vs. capacity allocation models**
  - Physical flow models represent the relationships between changes in pressure, flow, temperature within the natural gas pipeline network. Reflect engineering constraints on allowed pressure and compressor capabilities
    - Transient
    - Steady-state
  - Capacity allocation models represent re-allocation of pipeline transmission capacity between certain receipt and delivery points. Capacity is a construct reflecting daily transfer under "design" conditions. Determined using physical models.
- **Simulation vs. Optimization**
  - Simulation models compute dynamics for transient, or statics for steady-state, changes in gas flow and pressure with given receipts, deliveries and compressor settings. Rely on PDE representation of gas dynamics in each pipe with common boundary conditions at connections
  - Optimization models can determine receipt and delivery schedules and/or compressor operations to optimize certain objective functions

Operational coordination and co-optimization of gas and electric systems require optimization tools based on physical flow transient models which until very recently were mathematically intractable

Typical Simplified Representation of Pipeline Networks

Illustrative 25 pipe LANL Gas Network



- Network is represented as a graph
  - Edges are pipes with or without boosters (compressors)
  - Nodes are pipe junctions
  - Supply and demand occur at nodes
- This is a reduced network
  - Compressor stations are networks of individual compressors
  - Actual networks contain parallel pipes, valves and other flow and pressure control devices
- Key variables
  - Inlet and outlet flows for pipes
  - Pressures at pipe ends and nodes
  - Compressor capacities and other characteristics

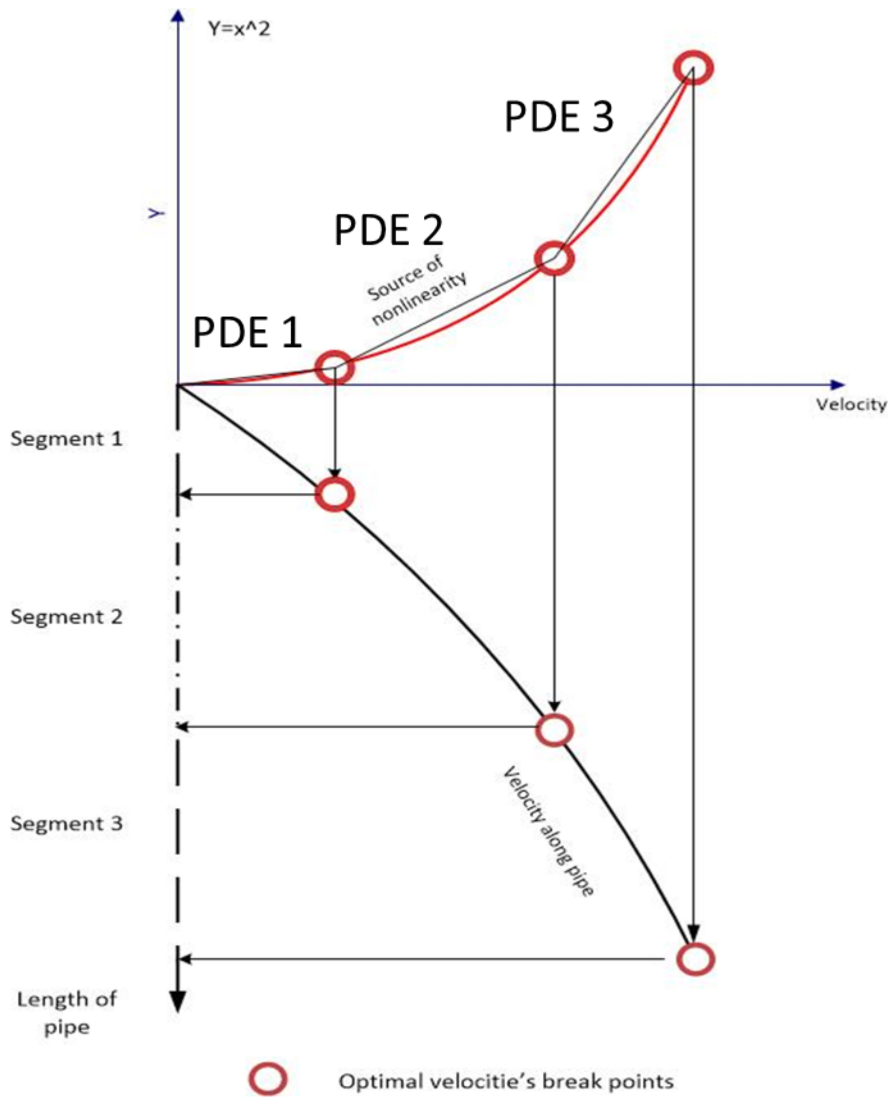
Generic Structure of the Natural Gas Pipeline Network Optimization Model

Model component	Description	Network variables, type
Objective function	Maximum Social Welfare of market transactions over the network or Maximum deliverability or Minimum demand shedding other	Flows. Instantaneous algebraic, typically linear
Nodal mass balance	Kirchhoff law on mass flow through each pipeline node	Flows. Instantaneous algebraic, linear
Nodal pressure balance	Equating pipe end pressures at a common node	Pressures. Instantaneous algebraic, linear
Nodal pressure constraints	Lower limits on gas pressure at network nodes	Pressures. Instantaneous algebraic, linear
In-pipe pressure constraints	Upper limits on the in-pipe pressure at discharge points of compressor station	Pressures. Instantaneous algebraic, linear
Transient flow equations	Relationships between pressures and flows within each pipe	Flows, pressures. PDEs. Non-linear
Compressor related constraints	Representation of the feasibility set for flows, inlet and outlet pressure	Flows, pressures. Instantaneous algebraic, non-linear
Other constraints	Supply and demand models, initial and terminal conditions	Flows, pressures. Instantaneous algebraic, linear

The Novel Piecewise Linearization Approach

- Involves a 5-step process that performed once to approximate a system of non-linear PDEs for each pipe with a system of linear algebraic equations
- The same process is applied to each pipe individually and is highly parallelizable
- Once the process is completed, the LP matrix is form and applicable to a wide range of system conditions and problems
- **Step 1**. Auxiliary segmentation of the pipe into several segments of smaller size (about 7 segments per 100 km)
- **Step 2**. Local approximation of PDEs for each pipe segment with linear PDEs transformed through variable substitution into the Klein-Gordon equation
- **Step 3**. Representation of the closed form Green's function for the Klein-Gordon equation using known trigonometric series
- **Step 4**. Computation of matrix coefficients connecting current end of pipe segment pressure to the history of in-flows and out-flows for the segment
- **Step 5**. A telescopic assembly process eliminating internal segment points and deriving coefficients directly for two ends of the pipe. NO DIMENSIONALITY INCREASE

Step 1. Pipe Segmentation



Continuity $$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho \nu)}{\partial x} = 0$$

Momentum $$\frac{\partial(\rho \nu)}{\partial t} + \frac{\partial p}{\partial x} = -\frac{\lambda \rho \boxed{v\,|\,v\,|}}{2D} - g\rho \sin\theta$$

State $$p = \rho ZRT = c^2 \rho$$

The number and size of segments are determined using special rules derived from the analysis of the solution of Klein-Gordon equation

Shorter pipes need no discretization

A 100 km pipe needs 7 - 10 segments

This discretization is only at the preparation stage. Optimization needs no pipe segmentation

Step 2. Linearized PDEs

For each segment we linearize the velocity term

$$v|v| \approx \kappa v + \chi$$

that leads to linear PDE for flows in each segment

$$\frac{\partial^2 \phi}{\partial t^2} + d\frac{\partial \phi}{\partial t} = c^2\frac{\partial^2 \phi}{\partial x^2} + s\frac{\partial \phi}{\partial x}.$$

Substitution

$$\phi(x,t) = exp\left(-\frac{s}{2c^2}x - \frac{d}{2}t\right)u(x,t)$$

leads to Klein- Gordon equation

$$\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2} - bu$$

- Green's function for a linear PDE is a response of the system to a simple instantaneous shock (Dirac's delta – function) applied at a certain place at a certain time

The Green's function of Klein- Gordon equation ($G_{kg}(x,\xi,t)$) is known. The impact of any set of initial and boundary conditions can be computed as a convolution of $G_{kg}(x,\xi,t)$) and these conditions

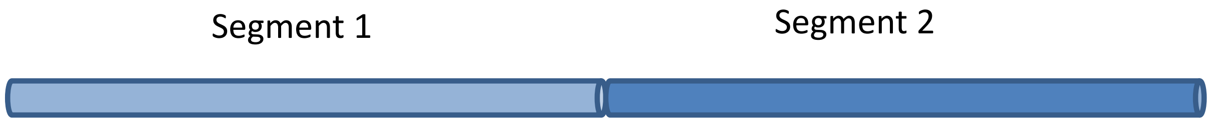
$$u(x,t) = \frac{\partial}{\partial t}\int_0^l \varphi_0(\xi)G_{kg}(x,\xi,t)d\xi$$
$$+\int_0^l \varphi_1(\xi)G_{kg}(x,\xi,t)d\xi$$
$$+c^2\int_0^t u(0,\tau)[\frac{\partial}{\partial \xi}G_{kg}(x,\xi,t-\tau)]|_{\xi=0}d\tau$$
$$-c^2\int_0^t u(l,\tau)[\frac{\partial}{\partial \xi}G_{kg}(x,\xi,t-\tau)]|_{\xi=l}d\tau$$

Steps 3 – 4: Green's Function –> Coefficients for Pipe Segments

- For each pipe's segment, Green's function is represented through known trigonometric series
- Special conditions on segment sized guarantee that series converge to a physically meaningful the solution. These conditions are used to determine the number and sizes of pipe segment
- When boundary conditions are discretized, application of the Green's function yields for each pipe linear algebraic relationships between pressures and flows at the end of each segment and initial conditions

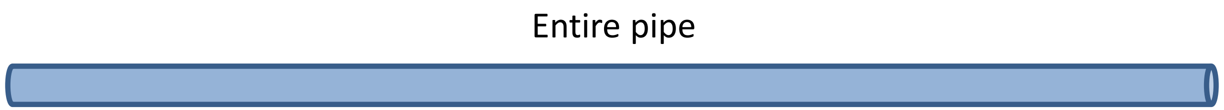
$$
\begin{aligned}
p_{0t} &= a_0 q_{0t} + a_1 q_{0t-1} + \ldots + a_k q_{0t-k} \\
&\quad - b_0 q_{lt} - b_1 q_{lt-1} - \ldots - b_k q_{lt-k} + r_{0k} V_{t-k} \\
p_{lt} &= c_0 q_{0t} + c_1 q_{0t-1} + \ldots + c_k q_{0t-k} \\
&\quad - d_0 q_{lt} - d_1 q_{lt-1} - \ldots - d_k q_{lt-k} + r_{lk} V_{t-k}
\end{aligned}
$$

Step 5: From Pipe Segmentation Back to the Entire Pipe



$$p_0 = A1_{11} q_0 - A1_{12} q_x + c1_1$$
$$p_x = A1_{21} q_0 - A1_{22} q_x + c1_2$$

$$p_x = A2_{11} q_x - A2_{12} q_l + c2_1$$
$$p_l = A2_{21} q_x - A2_{22} q_l + c2_2$$



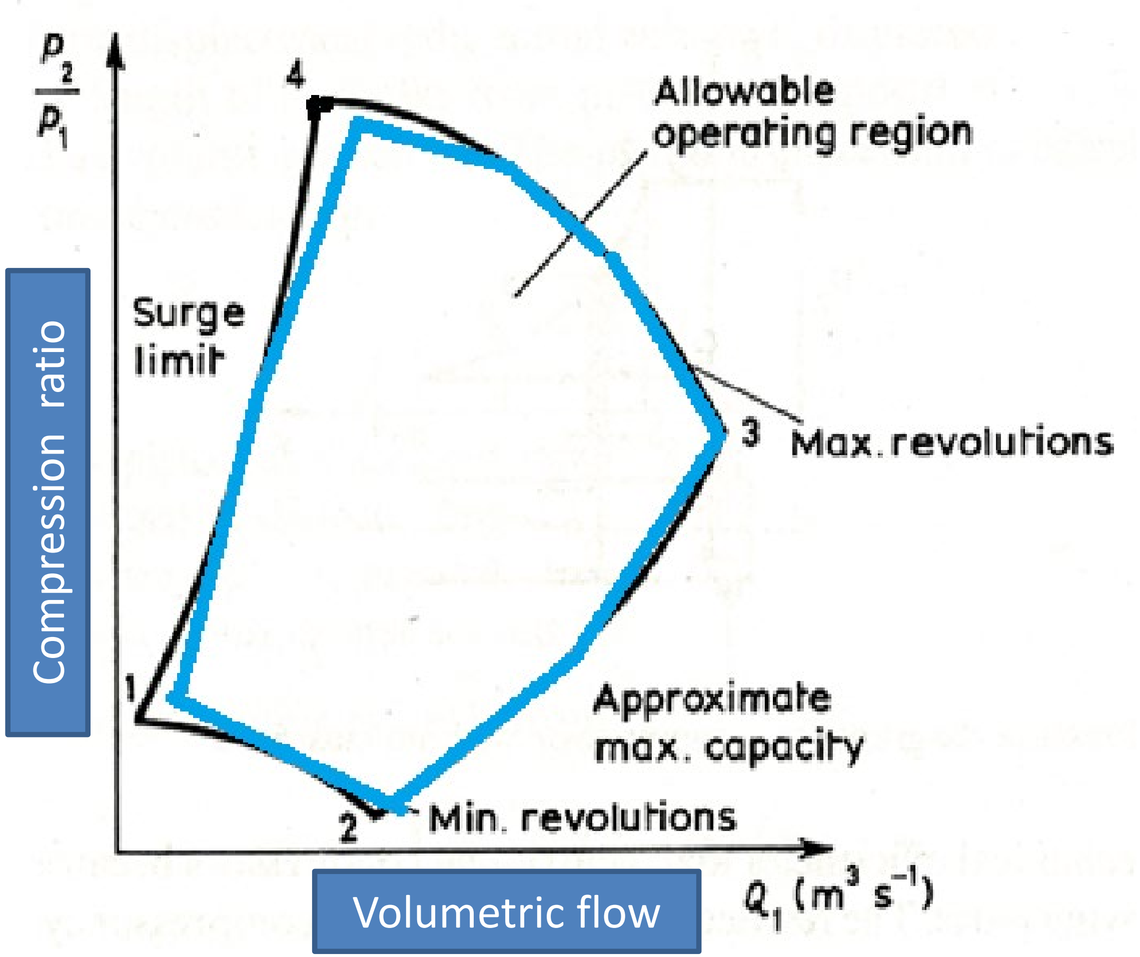
$$p_0 = B_{11} q_0 - B_{12} q_l + D_1 \qquad p_l = B_{21} q_0 - B_{22} q_l + D_2$$

$$B_{11} = A1_{11} - A1_{12} S A1_{21}$$
$$B_{12} = A1_{12} S A2_{12}$$
$$D_1 = c1_1 - A1_{12} S (c1_2 - c2_1)$$
$$D_2 = c2_2 + A2_{21} S (c1_2 - c2_1)$$

$$S = (A1_{22} + A2_{11})^{-1}$$

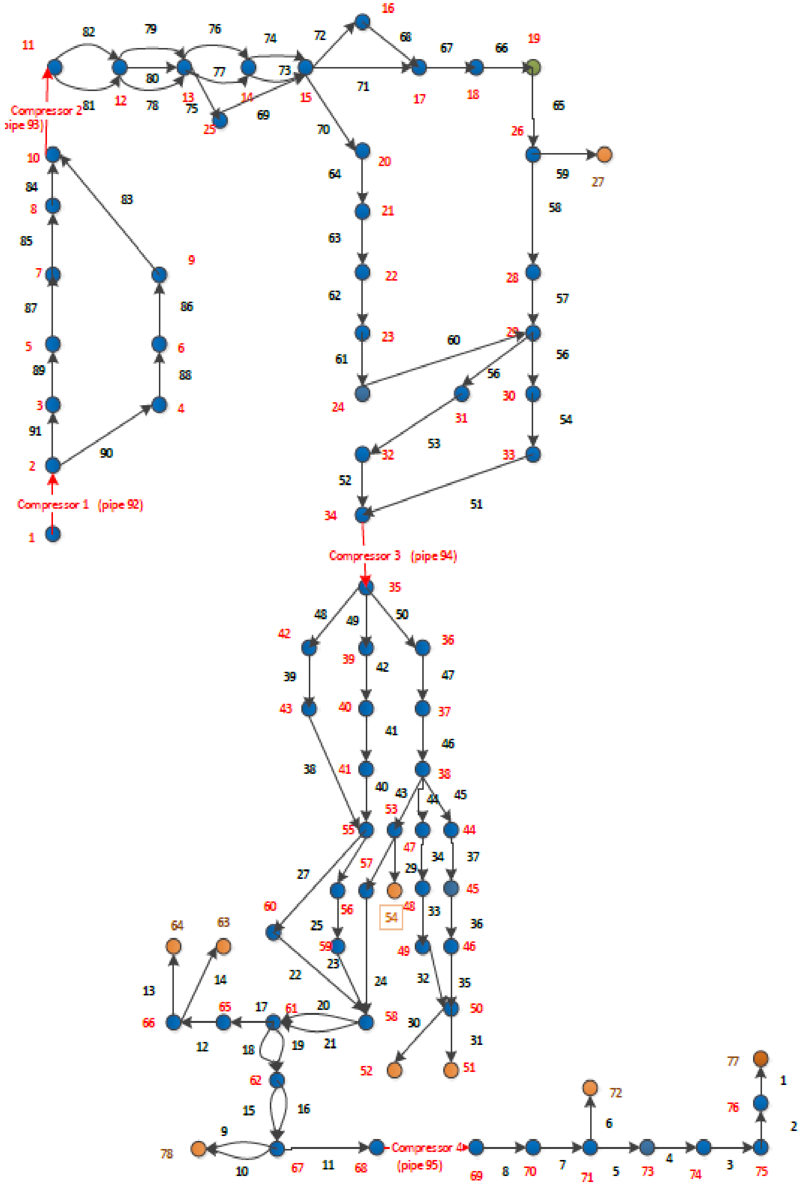
- Solutions for individual segments are formulaically merged at break-points to assure continuity of segmented solutions
- The continuity conditions translate into a system of linear equations for coefficients applicable to the entire pipe
- Solution to this linear system yields needed coefficients

Linearized Models of Centrifugal Compressors



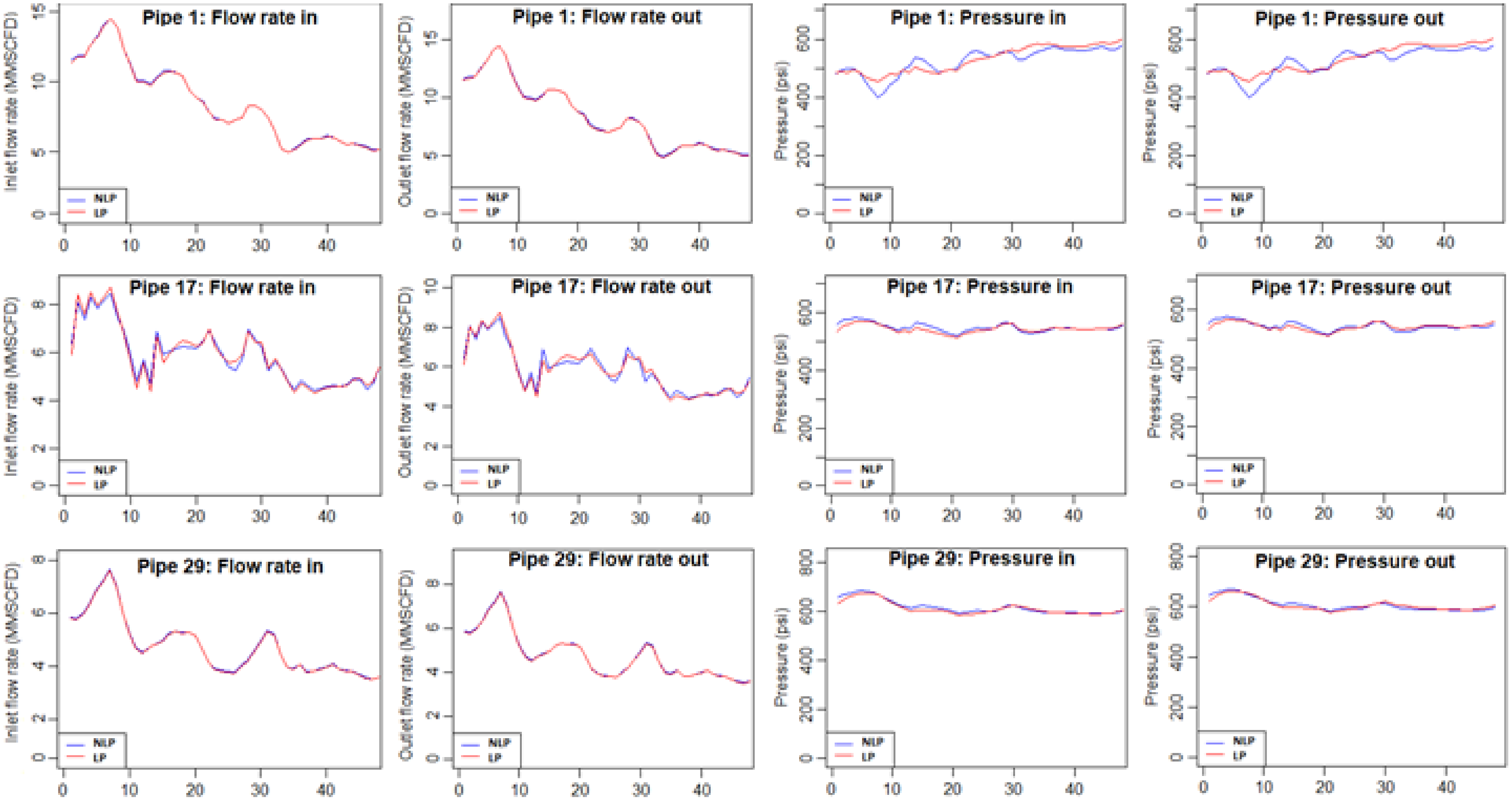
- Operative envelope: a feasibility area for the compression ratio and volumetric flow
- Linearization is obtained in the form of a convex polygon inscribed into the operative envelope of a compressor
- The latter is transformed to a set of linear inequalities in terms of pressure and a **mass** flow

Numerical Example: Testbed Network and Comparison Experiment

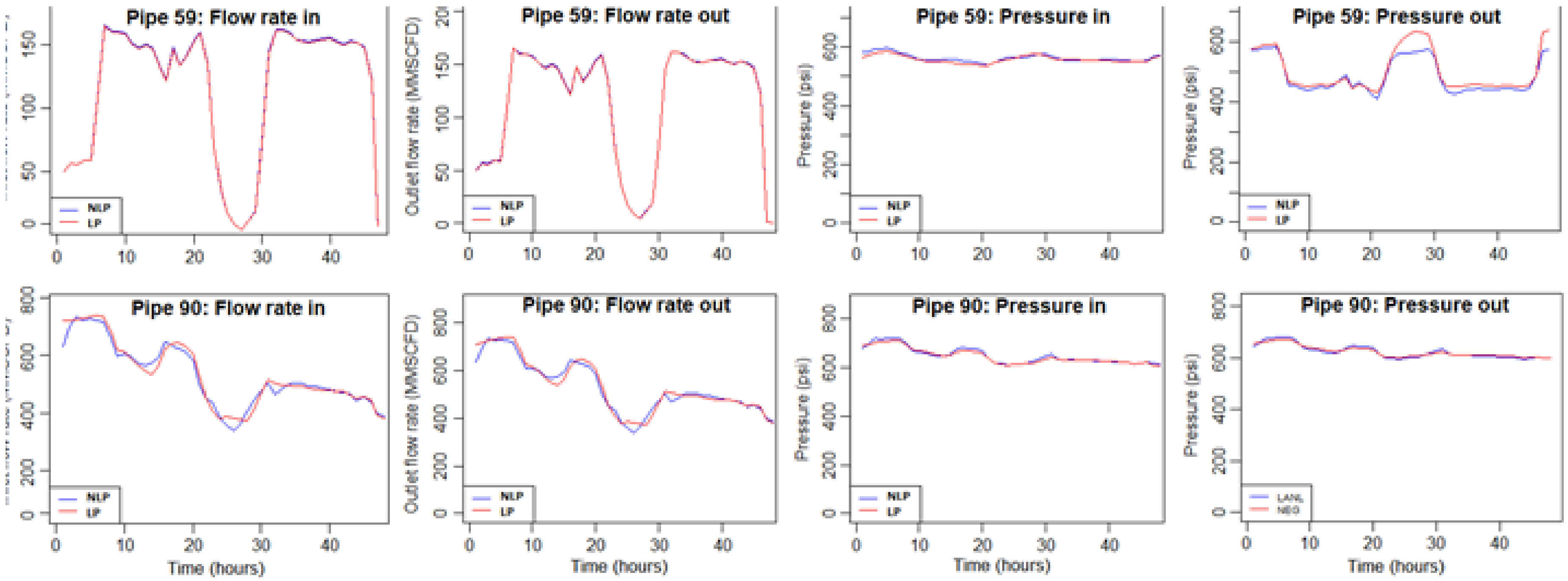


- Network is taken from the previous study and based on a segment of a real pipeline system. Data provided by Kinder Morgan
- The system is modeled under constrained conditions corresponding to Polar Vortex events
- The system was initially optimized using LANL's non-linear optimizer GRAIL adopted into GECO ENELYTIX as a gas network optimization module. Solution time was approximately 200 sec for a 48 hour horizon.
- Next the system was linearized using the above described method. Using the same computer, the corresponding LP was solved using barrier method in GUROBI solver. Solution time was 1.5 sec for a 48 hours horizon.

Numerical Example: Precision Comparison



Numerical Example: Precision Comparison (cont’d)



The proposed novel method demonstrates good precision within a wide range of pressure and flow values under nearly constrained conditions

Next Steps

- Further system reduction and speed-up is possible by:
  - eliminating balance equations for internal nodes with no supply and/or demand (similar to shift factor method for electric networks) and
  - monitoring pressures for location on the “watch list” only (lazy constraints approach)
- Further precision and comparisons with non-linear methods are needed
- Improved modeling of reciprocating compressors
- Adding models of other control devices
- Introducing integer variables with controlled valves and topology changes

Opportunities not to be missed

- Capability to perform fast transient modeling and optimization computations open the door for a variety of applications, such as:
  - Optimal dynamic scheduling of flows over large-scale networks
  - Static and dynamic optimization of compressor stations configuration
  - Optimization and modeling support for market mechanisms with granular pricing
  - System reliability assessments using Monte Carlo simulations
  - Combined gas-electric reliability assessments using coherent Monte-Carlo simulations of weather driven and other random events
  - Other applications we have not yet conceived

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