The Unit Commitment Problem: Convex Hulls and Strong Valid Inequalities

Yongpei Guan

Department of Industrial and Systems Engineering
University of Florida

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Joint work with Kai Pan

Overview

Overview

Power Systems

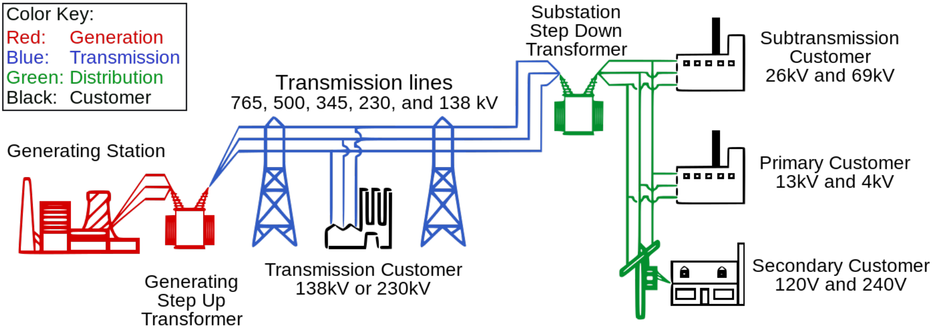
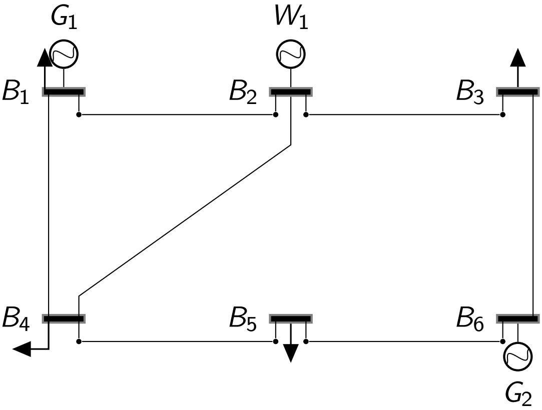


Figure: Basic Structure of Electric Power Grid[1]

Our focus: Transmission level

[1] Source: http://www.ferc.gov/industries/electric/indus-act/blackout/09-06-final-report.pdf

A Graph Representation



- Nodes $\rightarrow$ buses (generators and/or loads).
- Edges $\rightarrow$ transmission lines.

Electricity Markets

- Regulated market: Vertically Integrated Utilities.
- Deregulated market: Independent System Operators.

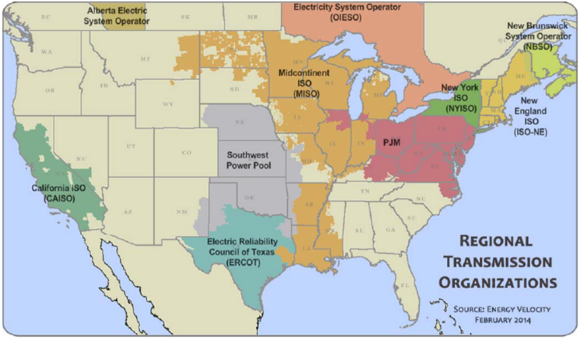


Figure: North American ISO Operating Regions

Over 3,000 utilities and more than 1,000 independent power producers.

Unit Commitment

Power System Operators

- Regulated electricity markets: Vertically Integrated Utilities.
  - Arizona Public Service, Florida Power & Lights, etc.
- Deregulated electricity markets: Independent Systems Operators.
  - California ISO, ERCOT, Midcontinential ISO, New York ISO, PJM, etc.

Objective: to minimize the total cost.

Market Participants

- Manage power generation assets and submit bids (offers) to participate in the wholesale electricity market.

Objective: to maximize the total profit.

Notation

- Parameters
  - $\overline{C}(\underline{C})$: capacity upper(lower) bound.
  - $L(\ell)$: minimum-up(-down) time limit.
  - V : stable ramp-up/-down rate limit.
  - $\overline{V}$: start-up/shut-down ramp rate limit.
- Decision Variables
  - y : on/off status (binary).
  - u : start-up operation (binary).
  - x : generation amount (continuous).

MIP UC Formulation

$$\min_{x,y,u} \quad \sum_{g=1}^{G}\Big(\sum_{t=1}^{T} f^g(x_t^g) + \sum_{t=2}^{T}\big(\mathrm{SU}^g u_t^g + \mathrm{SD}^g(y_{t-1}^g - y_t^g + u_t^g)\big)\Big)$$

s.t. Physical Constraints for Single Generator g, $\forall g \in [1, G]_{\mathbb{Z}}$,

$$\sum_{g=1}^{G} x_t^g = \sum_{b=1}^{B} d_t^b, \quad \forall t \in [1, T]_{\mathbb{Z}},$$

$$\sum_{g=1}^{G} \overline{C}_g y_t^g \geq (1 + r_t)\sum_{b=1}^{B} d_t^b, \quad \forall t \in [1, T]_{\mathbb{Z}},$$

$$-C_{jh} \leq \sum_{b=1}^{B} K_{jh}^b \Big(\sum_{g=1}^{G_b} x_t^g - d_t^b\Big) \leq C_{jh}, \quad \forall t \in [1, T]_{\mathbb{Z}}, \forall (j,h) \in \mathcal{E},$$

$$y_t^g \in \{0,1\},\ \forall t \in [1, T]_{\mathbb{Z}};\ u_t^g \in \{0,1\},\ \forall t \in [2, T]_{\mathbb{Z}}, \forall g \in [1, G]_{\mathbb{Z}}.$$

Model for power system operators

MIP UC Formulation

$$\max_{x,y,u} \quad \sum_{g=1}^{G}\Big(\sum_{t=1}^{T}\big(p_t x_t^g - f^g(x_t^g)\big) - \sum_{t=2}^{T}\big(\mathrm{SU}^g u_t^g + \mathrm{SD}^g(y_{t-1}^g - y_t^g + u_t^g)\big)\Big)$$

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$$y_t^g \in \{0, 1\}, \ \forall t \in [1, T]_{\mathbb{Z}}; \ u_t^g \in \{0, 1\}, \ \forall t \in [2, T]_{\mathbb{Z}}, \forall g \in [1, G]_{\mathbb{Z}}.$$

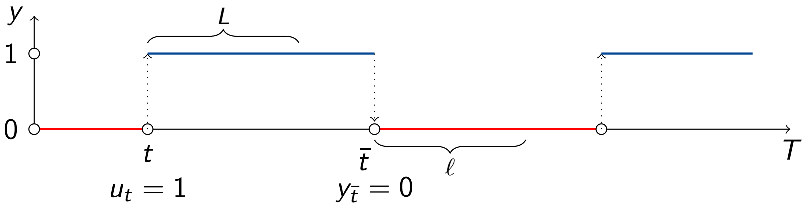
Model for market participants

Physical Constraints

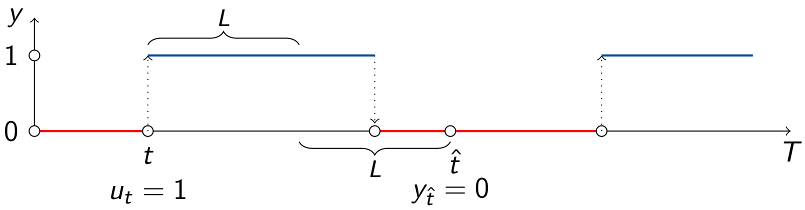
$$P := \Big\{ (x, y, u) \in \mathbb{R}_+^T \times \mathbb{B}^T \times \mathbb{B}^{T-1} :$$

logical	$y_t - y_{t-1} - u_t \leq 0, \ \forall t \in [2, T]_{\mathbb{Z}},$	(2)
lower bound	$x_t - \underline{C} y_t \geq 0, \ \forall t \in [1, T]_{\mathbb{Z}},$	(3)
upper bound	$x_t - \overline{C} y_t \leq 0, \ \forall t \in [1, T]_{\mathbb{Z}},$	(4)
ramp-up	$x_t - x_{t-1} \leq V y_{t-1} + \overline{V}(1 - y_{t-1}), \ \forall t \in [2, T]_{\mathbb{Z}},$	(5)
ramp-down	$x_{t-1} - x_t \leq V y_t + \overline{V}(1 - y_t), \ \forall t \in [2, T]_{\mathbb{Z}},$	(6)
minimum-up	$\sum_{i=t-L+1}^{t} u_i \leq y_t, \ \forall t \in [L+1, T]_{\mathbb{Z}},$	(7)
minimum-down	$\sum_{i=t-\ell+1}^{t} u_i \leq 1 - y_{t-\ell}, \ \forall t \in [\ell+1, T]_{\mathbb{Z}} \Big\}.$	(8)

Minimum-Up/-Down Time Constraints



Minimum-Up/-Down Time Constraints



Related Literature

Deterministic Unit Commitment Problem

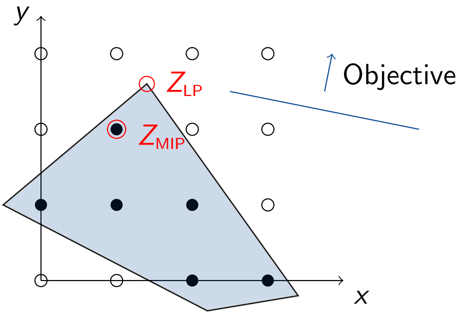
- Dynamic programming algorithm (Lowery 1966).
- Lagrangian relaxation (Muckstadt and Koenig 1977).
- Min-up/-down time polytope (Lee, Leung and Margot 2004).
- Min-up/-down time polytope with start-up cost (Rajan and Takriti 2005).
- Production ramping polytope (Damcı-Kurt, Küçükyavuz, Rajan, and Atamtürk 2014).

Our Study

- Polyhedral study by considering the polytope including minimum-up/-down time, ramping, and capacity constraints, i.e., Polytope P.

Cutting Planes

Shaping the linear feasible region to arrive from vertex Z_{LP} to Z_{MIP}



To prove optimality, Z_{MIP} must become a vertex by

- Branch and bound;

Cutting Planes

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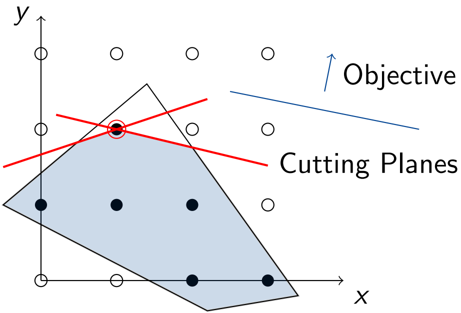
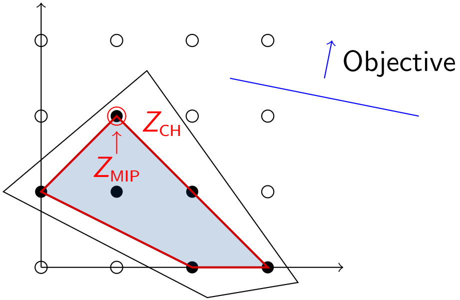


To prove optimality, Z_{MIP} must become a vertex by

- Branch and bound;
- and/or by adding cutting planes.

Convex Hull

The smallest convex feasible region containing all the feasible integer solutions



- The convex hull problems solve an MIP as an LP.
  - Each vertex is integral.
  - LP optimum is MIP optimum.

Overview

Three-Period Convex Hulls

- Case 1: $\underline{C} \leq \overline{V} < \underline{C} + V$, $\overline{C} - \overline{V} - V \geq 0$ (which implies $\overline{C} - \underline{C} - V \geq 0$); (the most common case for thermal generators)
- Case 2: $\underline{C} \leq \overline{V} < \underline{C} + V$, $\overline{C} - \underline{C} - V \geq 0$, and $\overline{C} - \overline{V} - V < 0$;
- Case 3: $\underline{C} \leq \overline{V} < \underline{C} + V$, $\overline{C} - \underline{C} - V < 0$ (which implies $\overline{C} - \overline{V} - V < 0$);
- Case 4: $\overline{V} \geq \underline{C} + V$, $\overline{C} - \overline{V} - V \geq 0$ (which implies $\overline{C} - \underline{C} - V \geq 0$);
- Case 5: $\overline{V} \geq \underline{C} + V$, $\overline{C} - \underline{C} - V \geq 0$, and $\overline{C} - \overline{V} - V < 0$.

Three-Period Convex Hulls

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Minimum-up/-down time

1. $L = \ell = 1$
2. $L = 1,\ \ell = 2$
3. $L = 2,\ \ell = 1$
4. $L = \ell = 2$

Parameter

1. $\overline{C} - \underline{C} - 2V < 0$
2. $\overline{C} - \underline{C} - 2V \geq 0$

$L = \ell = 2$ and $\overline{C} - \underline{C} - 2V \geq 0$

Original polytope:

$$P_3^2 := \Big\{ (x, y, u) \in \mathbb{R}_+^3 \times \mathbb{B}^3 \times \mathbb{B}^2 :$$

$$u_2 + u_3 \leq y_3, \tag{9a}$$

$$y_1 + u_2 + u_3 \leq 1, \tag{9b}$$

$$u_2 \geq y_2 - y_1, \ u_3 \geq y_3 - y_2, \tag{9c}$$

$$x_1 \geq \underline{C} y_1, \ x_2 \geq \underline{C} y_2, \ x_3 \geq \underline{C} y_3, \tag{9d}$$

$$x_1 \leq \overline{C} y_1, \ x_2 \leq \overline{C} y_2, \ x_3 \leq \overline{C} y_3, \tag{9e}$$

$$x_2 - x_1 \leq V y_1 + \overline{V}(1 - y_1), \ x_3 - x_2 \leq V y_2 + \overline{V}(1 - y_2), \tag{9f}$$

$$x_1 - x_2 \leq V y_2 + \overline{V}(1 - y_2), \ x_2 - x_3 \leq V y_3 + \overline{V}(1 - y_3) \Big\}. \tag{9g}$$

Strong Valid Inequalities

Proposition: For P_3^2, the following inequalities

$$x_1 \leq \overline{V} y_1 + V(y_2 - u_2) + (\overline{C} - \overline{V} - V)(y_3 - u_3 - u_2), \quad (10)$$

$$x_2 \leq \overline{V} y_2 + (\overline{C} - \overline{V})(y_3 - u_3 - u_2), \quad (11)$$

$$x_3 \leq \overline{C} y_3 - (\overline{C} - \overline{V}) u_3 - (\overline{C} - \overline{V} - V) u_2, \quad (12)$$

$$x_2 - x_1 \leq \overline{V} y_2 - \underline{C} y_1 + (\underline{C} + V - \overline{V})(y_3 - u_3 - u_2), \quad (13)$$

$$x_3 - x_2 \leq (\underline{C} + V) y_3 - \underline{C} y_2 - (\underline{C} + V - \overline{V}) u_3, \quad (14)$$

$$x_1 - x_2 \leq \overline{V} y_1 - (\overline{V} - V) y_2 - (\underline{C} + V - \overline{V}) u_2, \quad (15)$$

$$x_2 - x_3 \leq \overline{V} y_2 - \underline{C} y_3 + (\underline{C} + V - \overline{V})(y_3 - u_3 - u_2), \quad (16)$$

$$x_3 - x_1 \leq (\underline{C} + 2V) y_3 - \underline{C} y_1 - (\underline{C} + 2V - \overline{V}) u_3 - (\underline{C} + V - \overline{V}) u_2, \quad (17)$$

$$x_1 - x_3 \leq \overline{V} y_1 - \underline{C} y_3 + V(y_2 - u_2) + (\underline{C} + V - \overline{V})(y_3 - u_3 - u_2), \quad (18)$$

$$x_1 - x_2 + x_3 \leq \overline{V} y_1 - (\overline{V} - V) y_2 + \overline{V} y_3 + (\overline{C} - \overline{V})(y_3 - u_3 - u_2), \quad (19)$$

are valid for conv(P_3^2).

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$$x_1 - x_2 \leq \overline{V} y_1 - (\overline{V} - V) y_2 - (\underline{C} + V - \overline{V}) u_2, \quad (15)$$

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$$\Downarrow$$

$$\begin{aligned} x_1 &\le \overline{V} y_1 + (\overline{C} - \overline{V})(y_2 - u_2) \quad (0 \le y_2 - u_2 \le y_1) \\ &= \overline{V} y_1 + V(y_2 - u_2) + (\overline{C} - \overline{V} - V)(y_2 - u_2) \end{aligned}$$

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$$(0 \le y_3 - u_3 \le y_2)$$

Convex Hull

$$Q_3^2 := \Big\{ \quad (x, y, u) \in \mathbb{R}^8 : (9a) - (9d), (10) - (19), \\ u_2 \geq 0, \;\; u_3 \geq 0 \Big\}.$$

Convex Hull: Proof Sketch

Propositions:

1. Q_3^2 is full-dimensional.
2. Every inequality in Q_3^2 is valid for $\text{conv}(P_3^2)$, i.e., $\text{conv}(P_3^2) \subseteq Q_3^2$.
3. $Q_3^2 \subseteq \text{conv}(P_3^2)$.

   Prove $z = \sum_{s \in S} \lambda_s z^s$ for each $z \in Q_3^2$ where $z^s \in P_3^2$.

   - Discover a function $\lambda = f(y(z), u(z))$ such that $y(z) = \sum_{s \in S} \lambda_s y(z^s)$ and $u(z) = \sum_{s \in S} \lambda_s u(z^s)$.
   - Define a mapping $F(\lambda)$ from $P_3^2(y, u)$ to $Q_3^2(y, u)$.
   - Prove $x(z) = \sum_{s \in S} \lambda_s x(z^s)$, where $x(z) \in Q_3^2(y, u)$ and $x(z^s) \in P_3^2(y, u)$.

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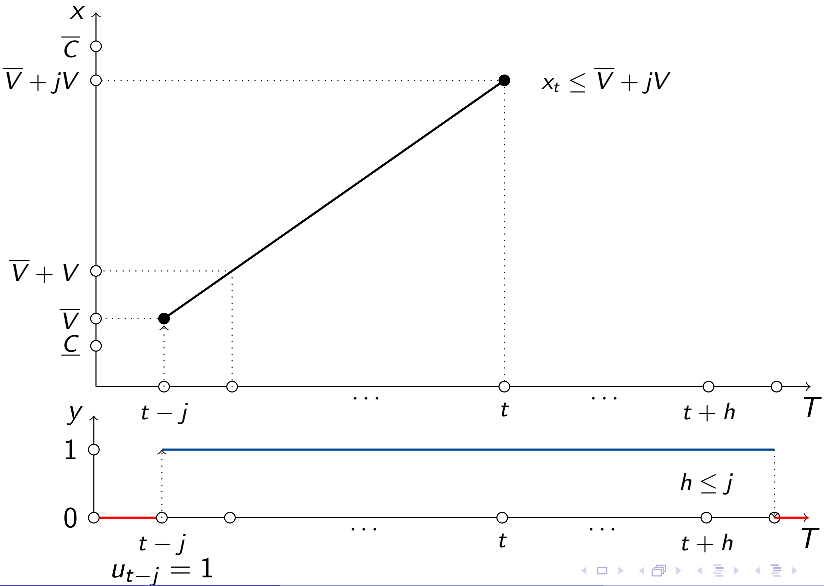
Innovative Proof!

Overview

Exponential-Sized Inequalities

Consider x_t:



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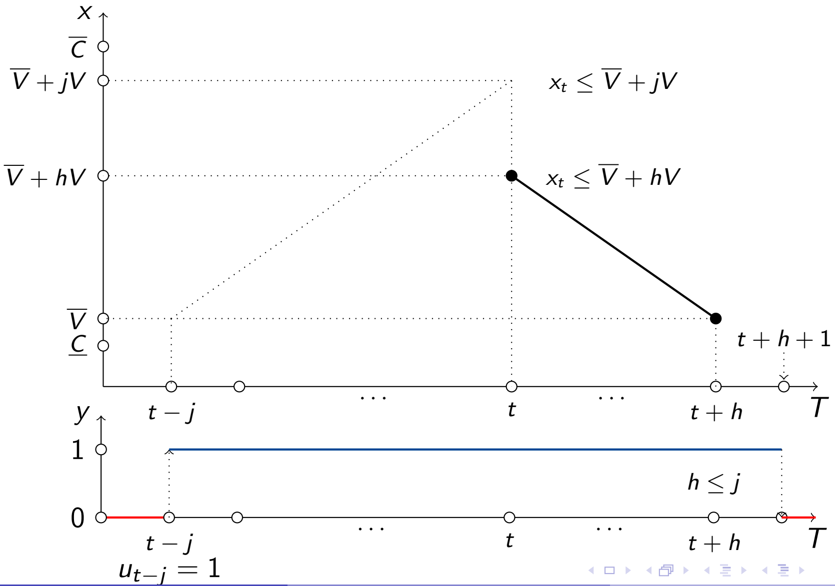

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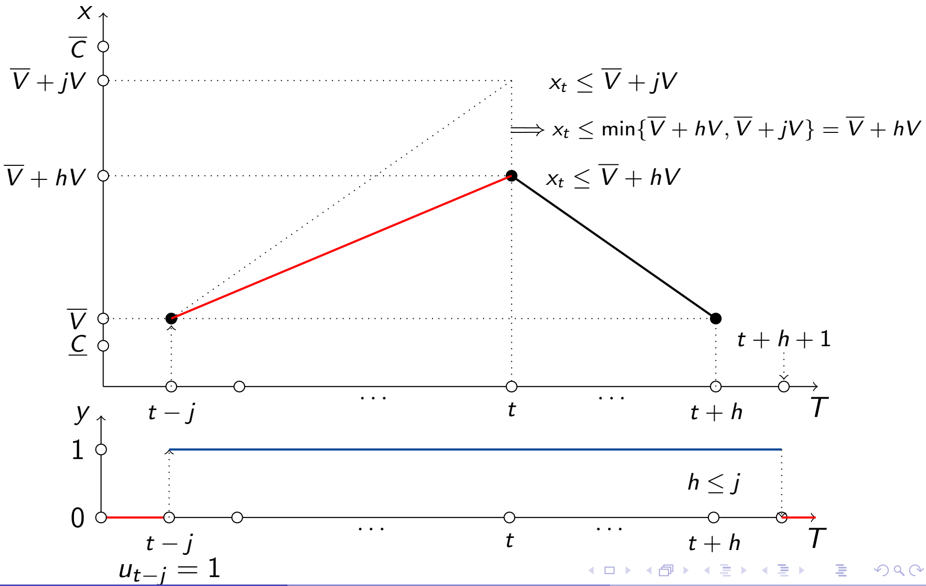

Proposition

For each $t \in T_1$, $j \in T_2$, $h \in T_3$, $S \subseteq [t-j+1, t]_{\mathbb{Z}}$, the inequality

$$x_t \leq \overline{V} y_t + V \sum_{i \in S} (i - d_i)(y_i - \sum_{m=0}^{L-1} u_{i-m}) + V \sum_{k=1}^{(h-1)^+} (y_{t+k} - \sum_{m=0}^{L-1} u_{t+k-m})$$
$$+ f(j,h) V (y_{t+h} - \sum_{m=0}^{L-1} u_{t+h-m}) + \psi(j)(y_{t-j} - \sum_{m=0}^{L-1} u_{t-j-m}) + \phi \tag{20}$$

is valid and facet-defining for conv(P), where $\phi = V \sum_{k=1}^{\min\{t+L-T-1, L-1\}} k u_{t-k}$
$+ V \sum_{k=(t+L-T)^+}^{L-1} \min\{L-1-k, k\} u_{t-k}$.

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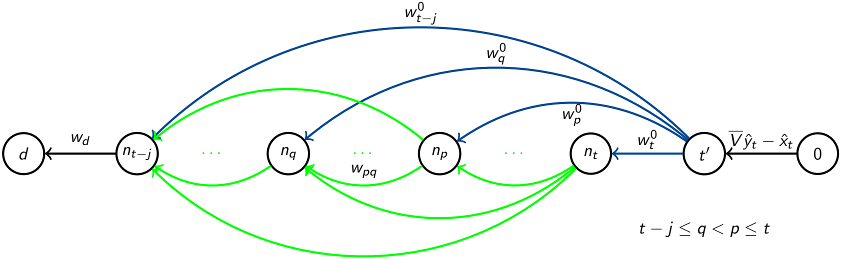
$+ V \sum_{k=(t+L-T)^+}^{L-1} \min\{L-1-k, k\} u_{t-k}$.

Separation

Input: a fractional solution $(\hat{x}, \hat{y}, \hat{u})$.
Set violation $vio \leftarrow 0$. Given t, j, h, construct a directed acyclic graph.



- The shortest path value obj, $vio \leftarrow obj$.

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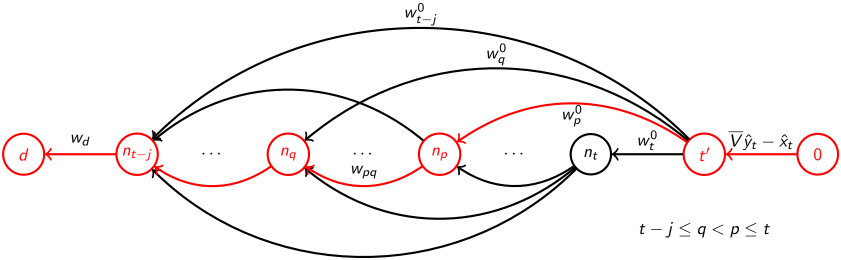

- If $vio < 0$, generate the inequality with $S = \{q, p\}$.

Proposition: Given a point $(\hat{x}, \hat{y}, \hat{u}) \in \mathbb{R}^{3n-1}_{+}$, there exists an $\mathcal{O}(T^3)$ time algorithm to find the most violated inequality (20), if any.

Convex Hulls for the Multi-Period Cases

Proposition

When $V = \overline{C} - \underline{C}$*, the following inequality*

$$x_t \leq \overline{V} y_t + (\overline{C} - \overline{V}) \left(y_s - \sum_{i=0}^{j} u_{s-i} \right), \ \forall t \in [1, T], \quad (21)$$

where $j = \min\{1, L-1, s-2, s-t\}$ *and* $s = \min\{t+1, T\}$*, is valid and facet-defining for conv(P).*

Convex Hulls for the Multi-Period Cases

Theorem

When $V = \overline{C} - \underline{C}$, the convex hull description for conv(P) can be described as follows:

$$P := \Big\{(x, y, u) \in \mathbb{R}_+^{3T-1} : \quad \sum_{i=t-L+1}^{t} u_i \le y_t, \ \forall t \in [L+1, T]_{\mathbb{Z}}, \tag{22a}$$

$$\sum_{i=t-\ell+1}^{t} u_i \le 1 - y_{t-\ell}, \ \forall t \in [\ell+1, T]_{\mathbb{Z}}, \tag{22b}$$

$$y_t - y_{t-1} - u_t \le 0, \ \forall t \in [2, T]_{\mathbb{Z}}, \tag{22c}$$

$$-x_t + \underline{C} y_t \le 0, \ \forall t \in [1, T]_{\mathbb{Z}}, \tag{22d}$$

$$\textit{Constraints } (21). \tag{22e}$$

Convex Hulls for the Multi-Period Cases

Proposition

For each $t \in [1, T]_{\mathbb{Z}}$, $m \in [0, \min\{[t - L - 1]^+, (\overline{C} - \overline{V})/V\}]_{\mathbb{Z}}$, $S \subseteq [t - m + 1, t - 1]_{\mathbb{Z}}$, *the inequality*

$$x_t \leq \overline{V} y_t + (L-1)V(y_t - \sum_{j=0}^{\min\{L-1,t-2\}} u_{t-j}) + V \sum_{i \in S \cup \{t\}} (i - d_i)(y_i - \sum_{j=0}^{\min\{L-1,i-2\}} u_{i-j})$$
$$+ (\overline{C} - \overline{V} - (m + L - 1)V)(y_{t-m} - \sum_{j=0}^{\min\{L-1,t-m-2\}} u_{t-m-j}) + V \sum_{j=0}^{\min\{L-1,t-2\}} j u_{t-j} \quad (23)$$

is valid and facet-defining for conv(P^U), where for each $i \in [t - m + 1, t]_{\mathbb{Z}}$, $d_i = \max\{a \in S \cup \{t - m\} : a < i\}$ *and if* $m = 0$, *then* $d_t = t$.

Convex Hulls for the Multi-Period Cases

Proposition

For each $t \in [1, T]_{\mathbb{Z}}$, $m \in [\min\{[T-t-1]^+, L-1\}, \min\{[T-t-1]^+, (\overline{C}-\overline{V})/V\}]_{\mathbb{Z}}$, $S \subseteq [t+L+1, t+m]_{\mathbb{Z}}$, *the inequality*

$$x_t \leq \overline{V} y_t + V \sum_{i=1}^{\min\{m,L-1\}} (y_{t+i} - \sum_{j=1}^{i} u_{t+j}) + V \sum_{i \in S \cup \{t+L\}} (d_i - i)(y_i - \sum_{j=0}^{\min\{m,L-1\}} u_{i-j})$$
$$+ (\overline{C} - \overline{V} - mV)(y_{t+m+1} - \sum_{j=0}^{\min\{m,L-1\}} u_{t+m+1-j}) \quad (24)$$

is valid and facet-defining for $conv(P^D)$*, where for each* $i \in [t+L, t+m]_{\mathbb{Z}}$, $d_i = \min\{a \in S \cup \{t+m+1\} : a > i\}$ *and if* $m \leq L-1$*, then* $d_{t+L} = t+L$*. Meanwhile, we let* $y_{T+1} = y_T$ *and* $u_{T+1} = 0$.

Convex Hulls for the Multi-Period Cases

Proposition

For each $t \in [2, T]_{\mathbb{Z}}$, $m \in [1, \min\{t-1, (\overline{C} - \underline{C})/V\}]_{\mathbb{Z}}$, $S_0 \subseteq [t - m + L, t - 1]_{\mathbb{Z}}$, $S = S_0 \cup \{t\}$, $q = \min\{a \in S\}$, $\delta = \min\{L-1, m-1\}$, *the inequality*

$$x_t - x_{t-m} \leq \overline{V} y_t - \underline{C} y_{t-m} + V \sum_{i \in S \setminus \{t-m+L\}} (i - d_i)(y_i - \sum_{j=0}^{\delta} u_{i-j}) + \delta V (y_t - \sum_{j=0}^{\delta} u_{t-j}) + (\underline{C} + V - \overline{V})(y_q - \sum_{j=0}^{\delta} u_{q-j}) + V \sum_{j=0}^{\delta} j u_{t-j}, \quad (25)$$

is valid and facet-defining for conv(P^U), *where for each* $i \in S$, $d_i = \max\{a \in S \cup \{t - m + L\} : a < i\}$ *and if* $m \leq L$, *then* $d_t = t$.

Convex Hulls for the Multi-Period Cases

Proposition

For each $t \in [1, T-1]_{\mathbb{Z}}$, $m \in [1, \min\{T-t, (\overline{C}-\underline{C})/V\}]_{\mathbb{Z}}$, $S_0 = [t+L+1, t+m]_{\mathbb{Z}}$, $S = S_0 \cup \{t+L\}$, $q = \min\{a \in S\}$, $\delta = \min\{L-1, m-1\}$, *the inequality*

$$x_t - x_{t+m} \leq \overline{V} y_t - \underline{C} y_{t+m} + V \sum_{i=1}^{\delta} (y_{t+i} - \sum_{j=1}^{i} u_{t+j}) + V \sum_{i \in S \setminus \{t+m\}} (d_i - i)(y_i - \sum_{j=0}^{\delta} u_{i-j})$$
$$+ (\underline{C} + V - \overline{V})(y_q - \sum_{j=0}^{\delta} u_{q-j}) \qquad (26)$$

is valid and facet-defining for conv(P^D*), where for each* $i \in S \setminus \{t+m\}$, $d_i = \min\{a \in S \cup \{t+m\} : a > i\}$ *and if* $m \leq L$*, then* $d_{t+L} = t+L$.

Convex Hulls for the Multi-Period Cases

Theorem

The convex hull description for the integrated minimum-up/-down time and ramping-up polytope is

$$conv(P^U) = Q^U := \Big\{ \; (x, y, u) \in \mathbb{R}^{3T-1} : (22a) - (22d), (23), (25), \\ u_t \geq 0, \; \forall t \in [2, T]_{\mathbb{Z}} \Big\}. \quad (27)$$

The convex hull description for the integrated minimum-up/-down time and ramping-down polytope is

$$conv(P^D) = Q^D := \Big\{ (x, y, u) \in \mathbb{R}^{3T-1} : (22a) - (22d), (24), (26), (27) \Big\}.$$

Overview

Basic Settings

Machine:

- A computer node with two AMD Opteron 2378 Quad Core Processors at 2.4GHz.
- 4GB addressable memory.

MIP commercial solver:

- IBM ILOG CPLEX 12.3 under default settings.
- One hour time limit per run.
- Single thread, conventional branch-and-cut search, no presolve.

Formulations:

- "MILP": Original formulation.
- "Strong": Original formulation plus our proposed strong valid inequalities.

Self-Scheduling UC

Eight single generators from the data in Ostrowski et al 2012

- For each generator, test three cases with the price varying between a given range randomly and report the average results.
- All polynomial-sized inequalities and a part of exponential-sized inequalities are added as constraints; all the remaining inequalities are added in the branch-and-cut root node.

Computational Performance

Gen	IGap (%)		CPU Time(s) (TGap (%))		# of Nodes		# of Cuts
	MILP	Strong	MILP	Strong	MILP	Strong	Strong
1	30.94	0.07	*** (3.62) [3] ‡	73	31157	0	0
2	38.88	0.12	*** (4.86) [3]	178.7	46947	0	0
3	36.44	0.08	*** (4.84) [3]	104.1	26640	0	0
4	35.08	0.1	*** (3.92) [3]	105.7	22841	0	0
5	31.29	0.32	*** (1.71) [3]	126.7	58398	0	0
6	58.33	0.34	*** (7.68) [3]	124.1	40190	0	0
7	49.86	0.19	*** (5.87) [3]	48.7	31366	0	0
8	79.6	0.17	*** (24.08) [3]	193.2	32211	0	144

‡ # of instances not solved to optimality.

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‡ # of instances not solved to optimality.

Network-Constrained UC

Modified IEEE-118 Bus System

- 54 generators, 118 buses, 186 transmission lines, and 91 load buses.
- Different instances with different load settings.
- For each instance, randomly generate three cases and report the average results.
- Strong valid inequalities are added in the first 100 branch-and-cut nodes.

Modified IEEE 118-Bus System

Inst	Load	IGap (%)		CPU Time (s)		TGap (%)		# of Nodes		# of Cuts
		MILP	Strong	MILP	Strong	MILP	Strong	MILP	Strong	Strong
1	[0.5d-0.7d]	1.59	0.77	*** [3] ‡	1572.1 [2]	0.37	0.08	40564	17518	1511
2	[0.7d-0.9d]	1.14	0.33	*** [3]	1249.8	0.13	0.00	32544	8927	1138
3	[0.9d-1.1d]	1.17	0.37	*** [3]	2283.6 [1]	0.23	0.05	23592	10576	1897
4	[1.1d-1.3d]	1.14	0.34	*** [3]	2199.6 [1]	0.26	0.08	26810	16891	2213
5	[1.3d-1.5d]	1.09	0.18	*** [3]	3155.6 [2]	0.10	0.06	94305	27337	3171
6	[1.5d-1.7d]	1.16	0.14	*** [3]	2470.4	0.11	0.00	56800	15305	2839
7	[1.7d-1.9d]	1.31	0.08	*** [3]	*** [3]	0.15	0.04	63380	49099	1945

‡ # of instances not solved to optimality.

Modified IEEE 118-Bus System

Inst	Load	IGap (%)		CPU Time (s)		TGap (%)		# of Nodes		# of Cuts
		MILP	Strong	MILP	Strong	MILP	Strong	MILP	Strong	Strong
1	[0.5d-0.7d]	1.59	0.77	*** [3] ‡	1572.1 [2]	0.37	0.08	40564	17518	1511
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7	[1.7d-1.9d]	1.31	0.08	*** [3]	*** [3]	0.15	0.04	63380	49099	1945

‡ # of instances not solved to optimality.

Summary of Our Contributions

- The integrated minimum-up/-down time and ramping polytope was investigated.
  - Convex hulls for the three-period polytopes under all possible settings.
  - Facets and convex hulls for the multi-period polytope.
- Innovative proofs to show the convex hull results.
- Efficient polynomial time separation algorithm for the multi-period facets.
- Extensive experiments to solve both the network-constrained and self-scheduling unit commitment problems, which verify the strength of all the proposed facets.