Parallel Temporal Decomposition for Improved Unit Commitment in Power System Production Cost Modeling

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FERC's 2017 Technical Conference

GMLC: Multi-Scale Production Cost Models

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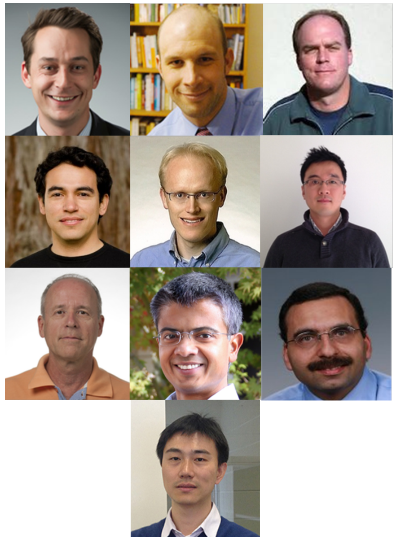
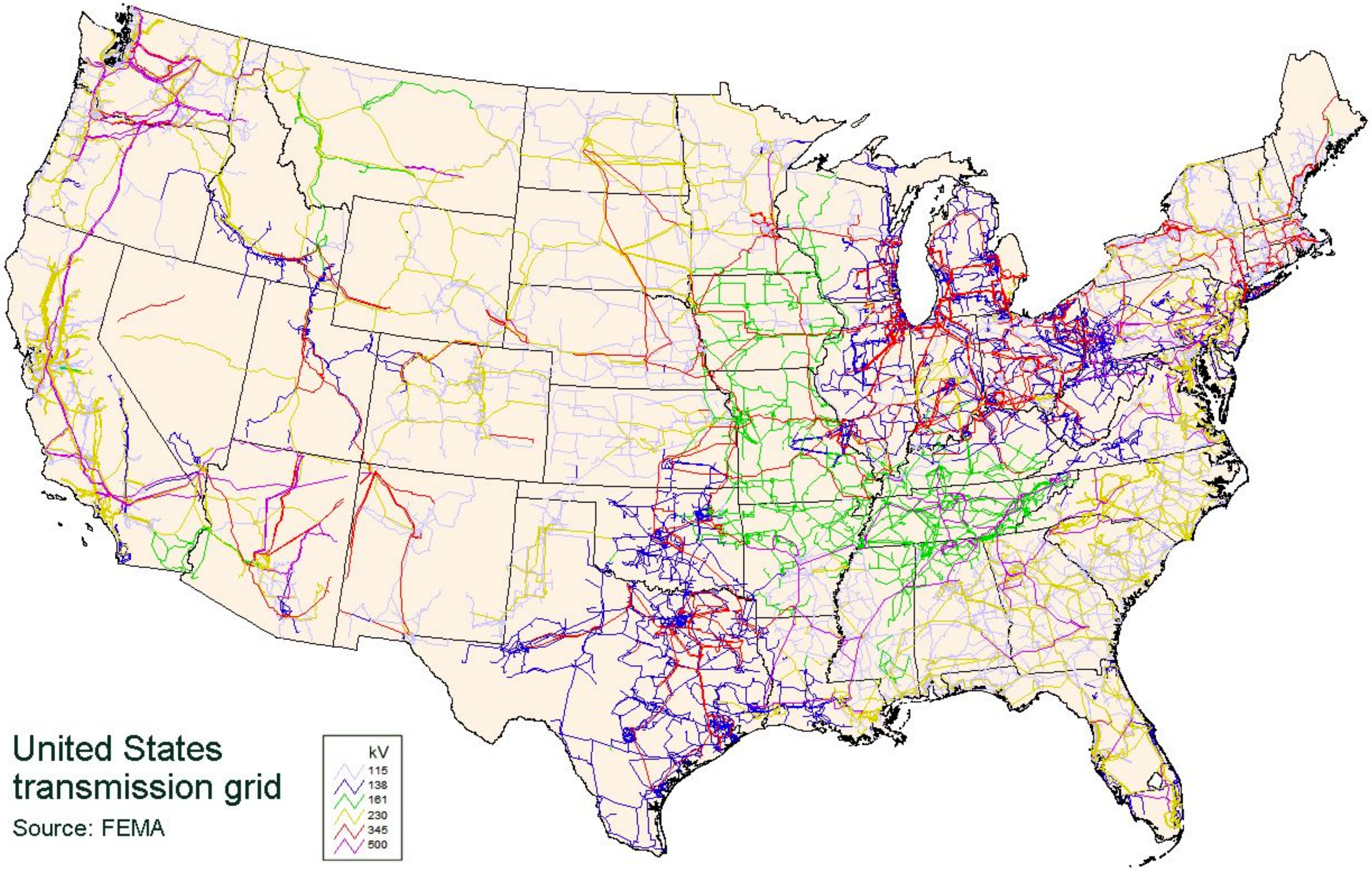
U.S. Department of Energy

- GMLC: Grid Modernization Laboratory Consortium
  - An aggressive five-year grid modernization strategy for the Department of Energy
- Design and planning tools sub-area includes Multi-Scale Production Cost Models
  - Develop multi-scale production cost models with faster mathematical solvers
- PCM Goal:
  - Substantially increase the ability of production cost models (PCM) to simulate power systems in more detail faster and more robustly.
  - Both Deterministic and Stochastic
- Talks at Technical Conference:
  - Session T1-B: Optimization Driven Scenario Grouping for Stochastic Unit Commitment (LLNL)
  - Session T2-B: Assessment of Wind Power Ramp Events in Scenario Generation for Stochastic Unit Commitment (SNL)
  - Session T3-A: Geographic Decomposition of Production Cost Models (NREL)
  - Session T3-A: Temporal Decomposition of the Production Cost Modeling in Power Systems (ANL)

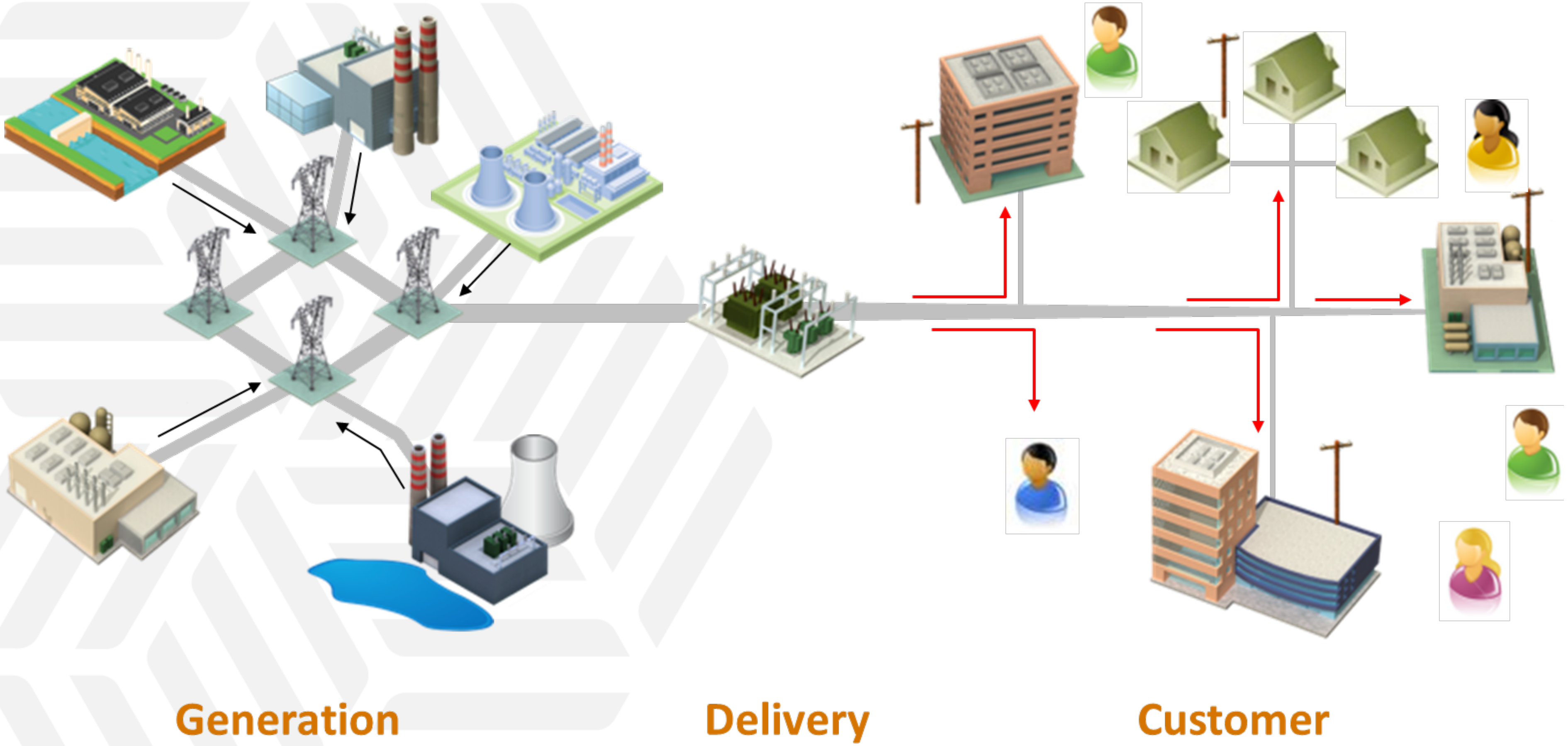




United States transmission grid

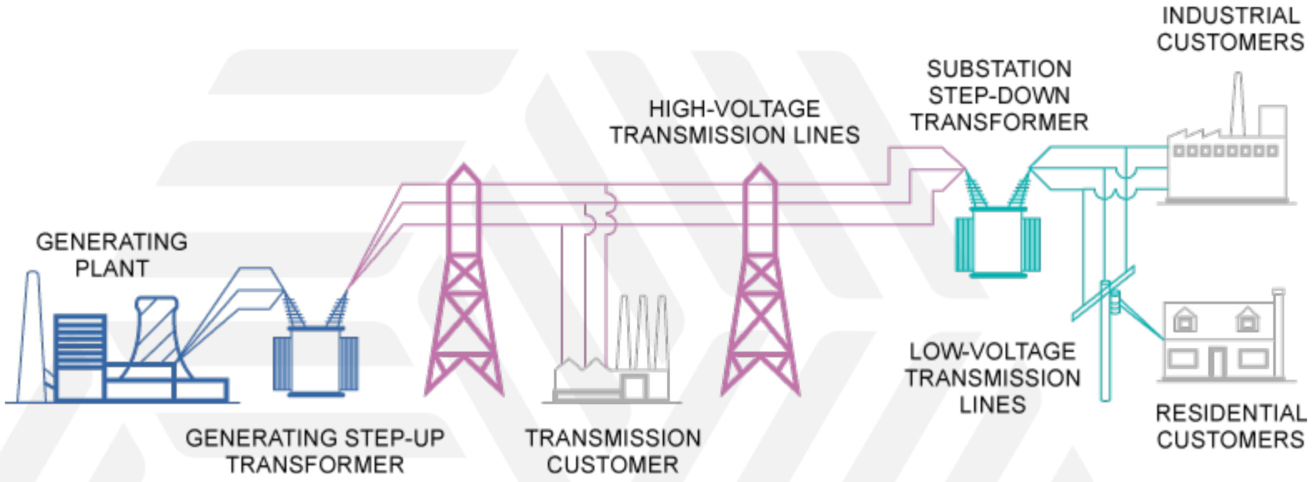
Source: FEMA

Power System Operations

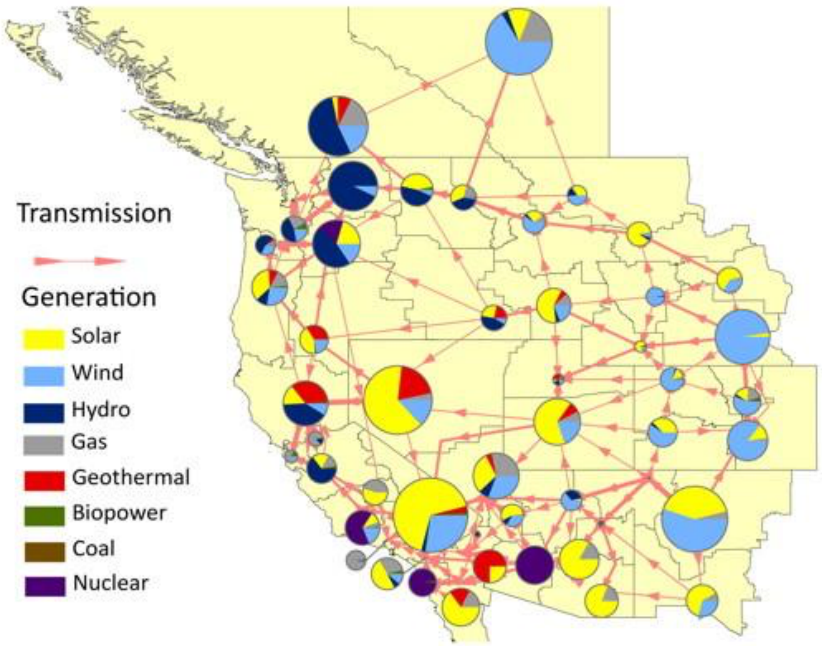


Source: EPRI, 2009

PCM: Unit Commitment and Economic Dispatch



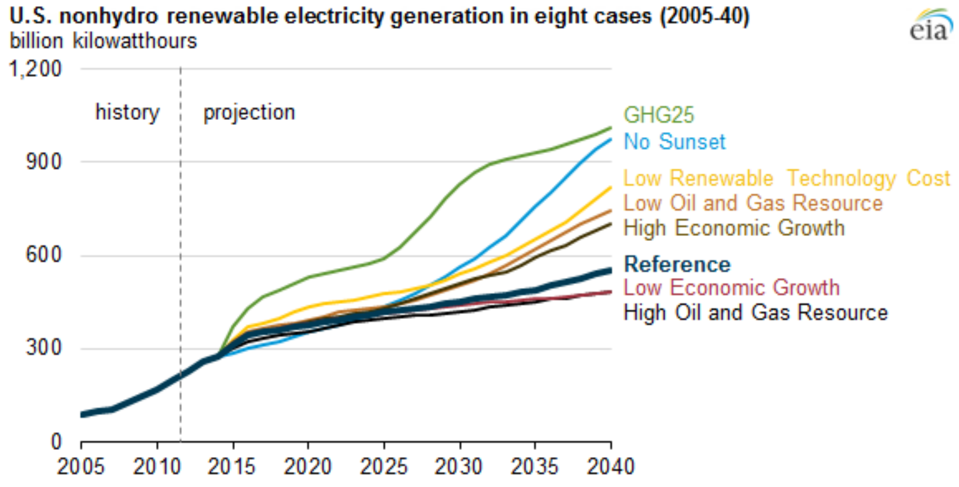
- ***Unit Commitment:*** scheduling generators on/off
- ***Economic Dispatch:*** scheduling power generation at each generator
- ***Security Constraints:***
  - Flow balance constraints
  - Power flow constraints
  - Ramping constraints
  - Minimum up/down constraints
  - Spinning reserve constraints

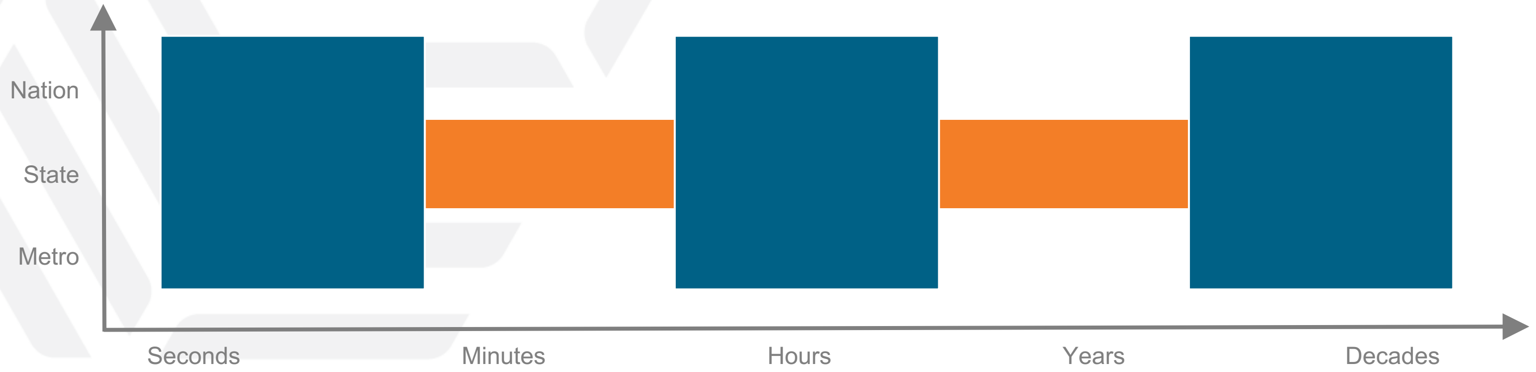


WECC System

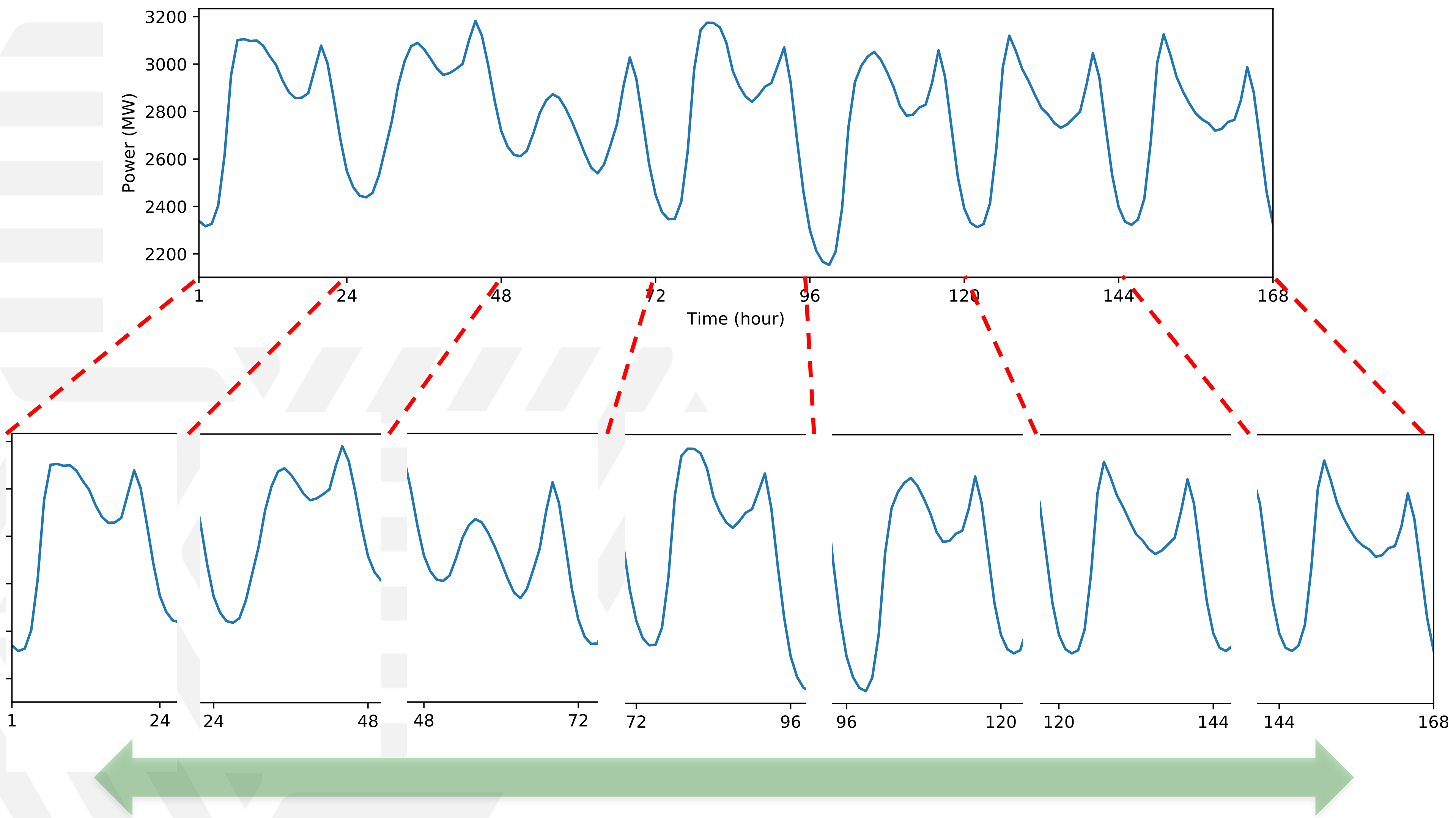
Multi-Scale Production Cost Modeling (PCM)

- **Goal**
  - to simulate a broad range of scenarios in order to plan electricity system over *a long-term planning horizon*
- **Challenges**
  - The complexity and resolution required to model the modern power system is rapidly increasing.
  - Model fidelity vs. execution time
  - Needs to solve long-term unit commitment and economic dispatch





Temporal Decomposition



Information is being shared between sub-horizons.

Long-Term UC Model Formulation

$\min \sum_{g\in G}\sum_{t\in T}(K_g u_{gt} + S_g v_{gt} + C_g p_{gt})$	Operating cost
s.t. $\sum_{l\in L_n^+} f_{lt} - \sum_{l\in L_n^-} f_{lt} + \sum_{g\in G_n} p_{gt} = D_{nt},\ n\in N,\ t\in T,$	Flow balance equation
$f_{lt} = B_l(\theta_{nt} - \theta_{mt}),\ l=(m,n)\in L,\ t\in T,$	Linearized power flow equation
$-F_l \le f_{lt} \le F_l,\ l\in L,\ t\in T,$	Transmission line capacity
$s_{gt} \le p_{gt} \le r_{gt},\ g\in G,\ t\in T,$	Operating reserve requirement
$r_{gt} \le P_g^{max} u_{gt},\ g\in G,\ t\in T,$ $s_{gt} \ge P_g^{min} u_{gt},\ g\in G,\ t\in T,$	Generation capacity
$r_{gt} - p_{g,t-1} \le R_g^+,\ g\in G,\ t\ge 2$ $s_{gt} - p_{g,t-1} \ge -R_g^-,\ g\in G,\ t\ge 2$	Ramping capacity
$\sum_{q=t-UT_g+1}^{t} v_{gq} \le u_{gt},\ g\in G,\ t\ge UT_g,$ $\sum_{q=t-DT_g+1}^{t} w_{gq} \le 1-u_{gt},\ g\in G,\ t\ge DT_g,$	Minimum uptime downtime requirements
$v_{gt} - w_{gt} = u_{gt} - u_{g,t-1},\ g\in G,\ t\ge 2,$ $u_{gt}, v_{gt}, w_{gt} \in \{0,1\},\ g\in G,\ t\in T$	Commitment logic

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Temporal Decomposition – Linking Constraints

$$\min \sum_{g \in G} \sum_{t \in T} (K_g u_{gt} + S_g v_{gt} + C_g p_{gt})$$

$$\text{s.t.} \sum_{l \in L_n^+} f_{lt} - \sum_{l \in L_n^-} f_{lt} + \sum_{g \in G_n} p_{gt} = D_{nt},\ n \in N,\ t \in T,$$

$$f_{lt} = B_l (\theta_{nt} - \theta_{mt}),\ l = (m,n) \in L,\ t \in T,$$

$$-F_l \le f_{lt} \le F_l,\ l \in L,\ t \in T,$$

$$s_{gt} \le p_{gt} \le r_{gt},\ g \in G,\ t \in T,$$

$$r_{gt} \le P_g^{max} u_{gt},\ g \in G,\ t \in T,$$

$$s_{gt} \ge P_g^{min} u_{gt},\ g \in G,\ t \in T,$$

$$r_{gt} - p_{g,t-1} \le R_g^+,\ g \in G,\ t \ge 2$$

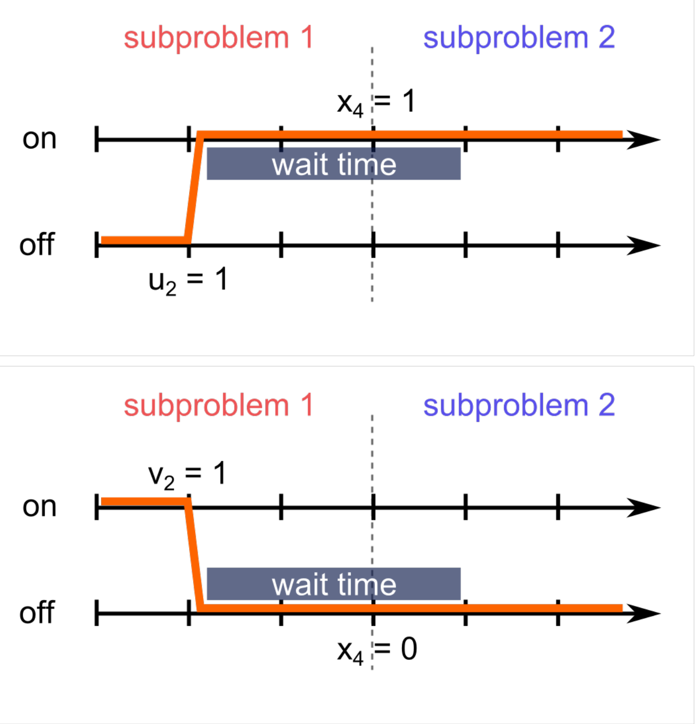
$$s_{gt} - p_{g,t-1} \ge -R_g^-,\ g \in G,\ t \ge 2$$

$$\sum_{q=t-UT_g+1}^{t} v_{gq} \le u_{gt},\ g \in G,\ t \ge UT_g,$$

$$\sum_{q=t-DT_g+1}^{t} w_{gq} \le 1 - u_{gt},\ g \in G,\ t \ge DT_g,$$

$$v_{gt} - w_{gt} = u_{gt} - u_{g,t-1},\ g \in G,\ t \ge 2,$$

$$u_{gt}, v_{gt}, w_{gt} \in \{0,1\},\ g \in G,\ t \in T$$



Coupling Constraints that link the variables in different time periods

Temporal Decomposition – Linking Constraints

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Subintervals of the time horizon:

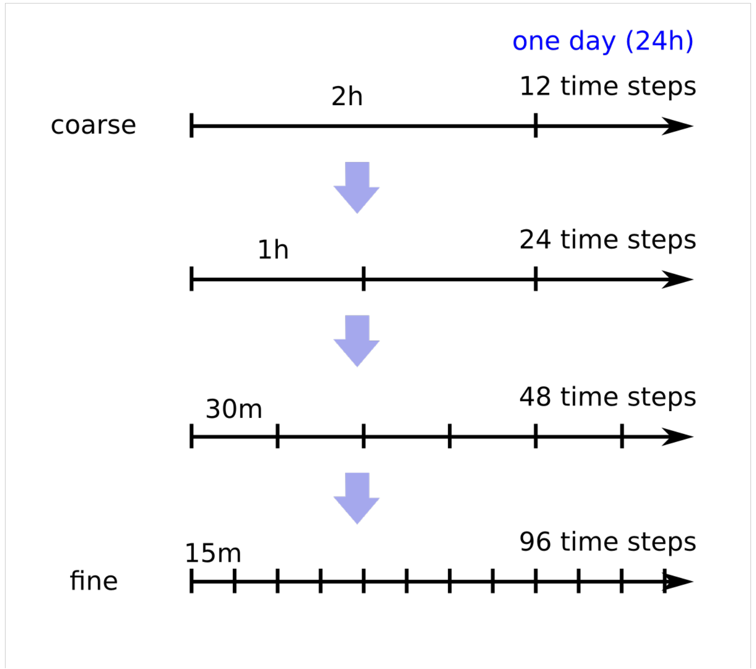
1. $T_j \subset T$ for $j \in J$;
2. $\cup_{j \in J} T_j = T$; and
3. $T_i \cap T_j = \emptyset$ for $i \neq j \in J$.

Equivalent PCM formulation:

$$\min \sum_{g \in G} \sum_{t \in T} (K_g u_{gt} + S_g v_{gt} + C_g p_{gt})$$

$$\begin{aligned}
\text{s.t.}\quad & r_{gt} - p_{g,t-1} \le R_g^+, \; g \in G, \; t \in T_j, \; t-1 \notin T_j, \; j \in J, \\
& s_{gt} - p_{g,t-1} \ge -R_g^-, \; g \in G, \; t \in T_j, \; t-1 \notin T_j, \; j \in J, \\
& \sum_{q=t-UT_g+1}^{t} v_{gq} \le u_{gt}, \; g \in G, \; t \in T_j, \; t - UT_g + 1 \notin T_j, \; j \in J, \\
& \sum_{q=t-DT_g+1}^{t} w_{gq} \le 1 - u_{gt}, \; g \in G, \; t \in T_j, \; t - DT_g + 1 \notin T_j, \; j \in J, \\
& v_{gt} - w_{gt} = u_{gt} - u_{g,t-1}, \; g \in G, \; t \in T_j, \; t-1 \notin T_j, \; j \in J, \\
& (\mathbf{u}_j, \mathbf{v}_j, \mathbf{w}_j, \mathbf{p}_j, \mathbf{r}_j, \mathbf{s}_j) \in \mathcal{X}_j, \; j \in J,
\end{aligned}$$

mixed-binary set



We simplify the formulation.

$$\begin{aligned}
\min\quad & \sum_{j \in J} \sum_{k \in K} \mathbf{c}_j \mathbf{x}_j \\
\text{s.t.}\quad & \sum_{j \in J} \sum_{k \in K} \mathbf{A}_j \mathbf{x}_j \ge \mathbf{b}, \\
& \mathbf{x}_j \in conv(\mathcal{X}_j), \; j \in J
\end{aligned}$$

Decomposition Methods

Simplified Original Formulation:

$$\begin{aligned} \min \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j \\ \text{s.t.} \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j \geq \mathbf{b}, \\ & \mathbf{x}_j \in conv(\mathcal{X}_j),\ j \in J \end{aligned}$$

Dantzig-Wolfe Decomposition

$$\begin{aligned} \min \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j^k \alpha_j^k \\ \text{s.t.} \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j^k \alpha_j^k \geq \mathbf{b}\ (\lambda), \\ & \sum_{k\in K} \alpha_j^k = 1\ (\mu_j),\ j \in J \\ & \alpha_j^k \geq 0,\ j \in J,\ k \in K, \end{aligned}$$

Dual Decomposition

$$\begin{aligned} \max_{\lambda\geq 0} \min \quad & \sum_{j\in J} \mathbf{c}_j \mathbf{x}_j + \lambda^T \left(\mathbf{b} - \sum_{j\in J} \mathbf{A}_j \mathbf{x}_j \right) \\ \text{s.t.} \quad & \mathbf{x}_j \in conv(\mathcal{X}_j),\ j \in J \end{aligned}$$

Dual Decomposition

- Outer approximation
- Row generation procedure
- Dual solution space
- Regularization

Dantzig-Wolfe Decomposition

- Inner approximation
- Column generation procedure
- Primal solution space
- Branching, heuristics

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Dual Decomposition

Simplified Original Formulation:

$$\begin{aligned} \min \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j \\ \text{s.t.} \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j \geq \mathbf{b}, \\ & \mathbf{x}_j \in conv(\mathcal{X}_j),\ j \in J \end{aligned}$$

Dual Decomposition

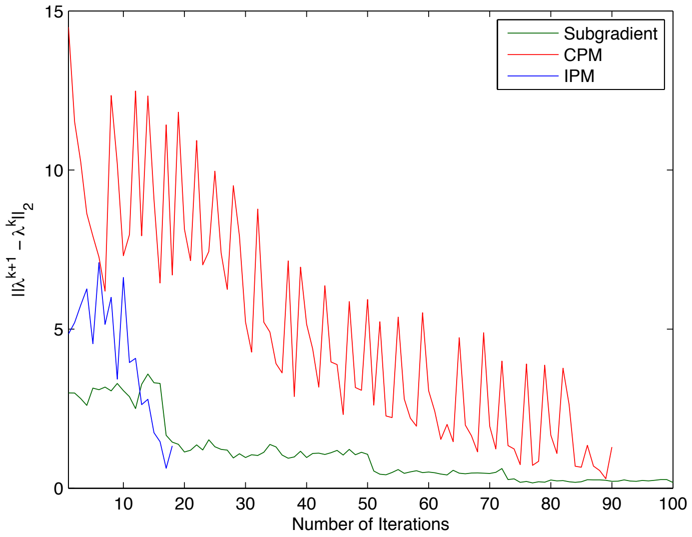
$$\begin{aligned} \max_{\lambda\geq 0} \min \quad & \sum_{j\in J} \mathbf{c}_j \mathbf{x}_j + \lambda^T \left(\mathbf{b} - \sum_{j\in J} \mathbf{A}_j \mathbf{x}_j \right) \\ \text{s.t.} \quad & \mathbf{x}_j \in conv(\mathcal{X}_j),\ j \in J \end{aligned}$$

Maximizing the Lagrangian dual bound

$$\max_{\lambda\geq 0} \left\{ \sum_{j\in J} D_j(\lambda) + \mathbf{b}^T \lambda \right\}$$

Decomposed for each *j*.

$$D_j(\lambda) := \min \left\{ \left(\mathbf{c}_j - \lambda^T \mathbf{A}_j \right) \mathbf{x}_j : \mathbf{x}_j \in conv(\mathcal{X}_j) \right\}$$



Proximal bundle model:

$$\begin{aligned} \max \quad & \sum_{j\in J} \mu_j + \mathbf{b}^T \lambda - \frac{1}{2} \|\lambda - \hat{\lambda}\|_2^2 \\ \text{s.t.} \quad & \mu_j \leq D_j(\lambda^k) - (\mathbf{A}_j \mathbf{x}_j)^T (\lambda - \lambda^k),\ j \in J,\ k \in K, \\ & \lambda \geq 0 \end{aligned}$$

Outer approximate

Dantzig-Wolfe Decomposition

Simplified Original Formulation:

$$\min \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j$$

$$\text{s.t.} \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j \ge \mathbf{b},$$

$$\mathbf{x}_j \in conv(\mathcal{X}_j),\ j \in J$$

Dantzig-Wolfe Decomposition

$$\min \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j^k \alpha_j^k$$

$$\text{s.t.} \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j^k \alpha_j^k \ge \mathbf{b}\ (\lambda),$$

$$\sum_{k\in K} \alpha_j^k = 1\ (\mu_j),\ j \in J$$

$$\alpha_j^k \ge 0,\ j \in J,\ k \in K,$$

Pricing Problems:

Add new column if

$$\mu_j^k > \min\left\{\left(\mathbf{c}_j - (\lambda^k)^T \mathbf{A}_j\right)\mathbf{x}_j : \mathbf{x}_j \in conv(\mathcal{X}_j)\right\}$$

- Decomposed for each j
- Same as the *dual decomposition subproblems*

Valid Lagrangian Dual Bound:

$$z_{DW} \ge \sum_{j\in J}\left(\mathbf{c}_j - (\lambda^k)^T \mathbf{A}_j\right)\mathbf{x}_j^k + \mathbf{b}^T \lambda^k$$

Best Lagrangian Dual Bound:

$$z \ge z_{DW} = \sum_{j\in J}\left(\mathbf{c}_j - (\lambda^k)^T \mathbf{A}_j\right)\mathbf{x}_j^k + \mathbf{b}^T \lambda^k$$

Positive duality gap may exist.

Integrating Branch-and-Bound Method

Simplified Original Formulation:

$$
\begin{aligned}
\min \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j \\
\text{s.t.} \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j \geq \mathbf{b}, \\
& \mathbf{x}_j \in conv(\mathcal{X}_j),\ j \in J
\end{aligned}
$$

Dantzig-Wolfe Decomposition

$$
\begin{aligned}
\min \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{c}_j \mathbf{x}_j^k \alpha_j^k \\
\text{s.t.} \quad & \sum_{j\in J}\sum_{k\in K} \mathbf{A}_j \mathbf{x}_j^k \alpha_j^k \geq \mathbf{b}\ (\lambda), \\
& \sum_{k\in K} \alpha_j^k = 1\ (\mu_j),\ j \in J \\
& \alpha_j^k \geq 0,\ j \in J,\ k \in K,
\end{aligned}
$$

Recovering Original Variables:

$$
\mathbf{x}_j = \sum_{k\in K} \mathbf{x}_j^k \alpha_j^k \in conv(\mathcal{X}_j)
$$

Convex combination of feasible solutions

- Not necessarily integer feasible

Branching in DW Decomposition:

$$\sum_{k\in K} \mathbf{x}_j^k \hat{\alpha}_j^k$$

Branching at fractional value

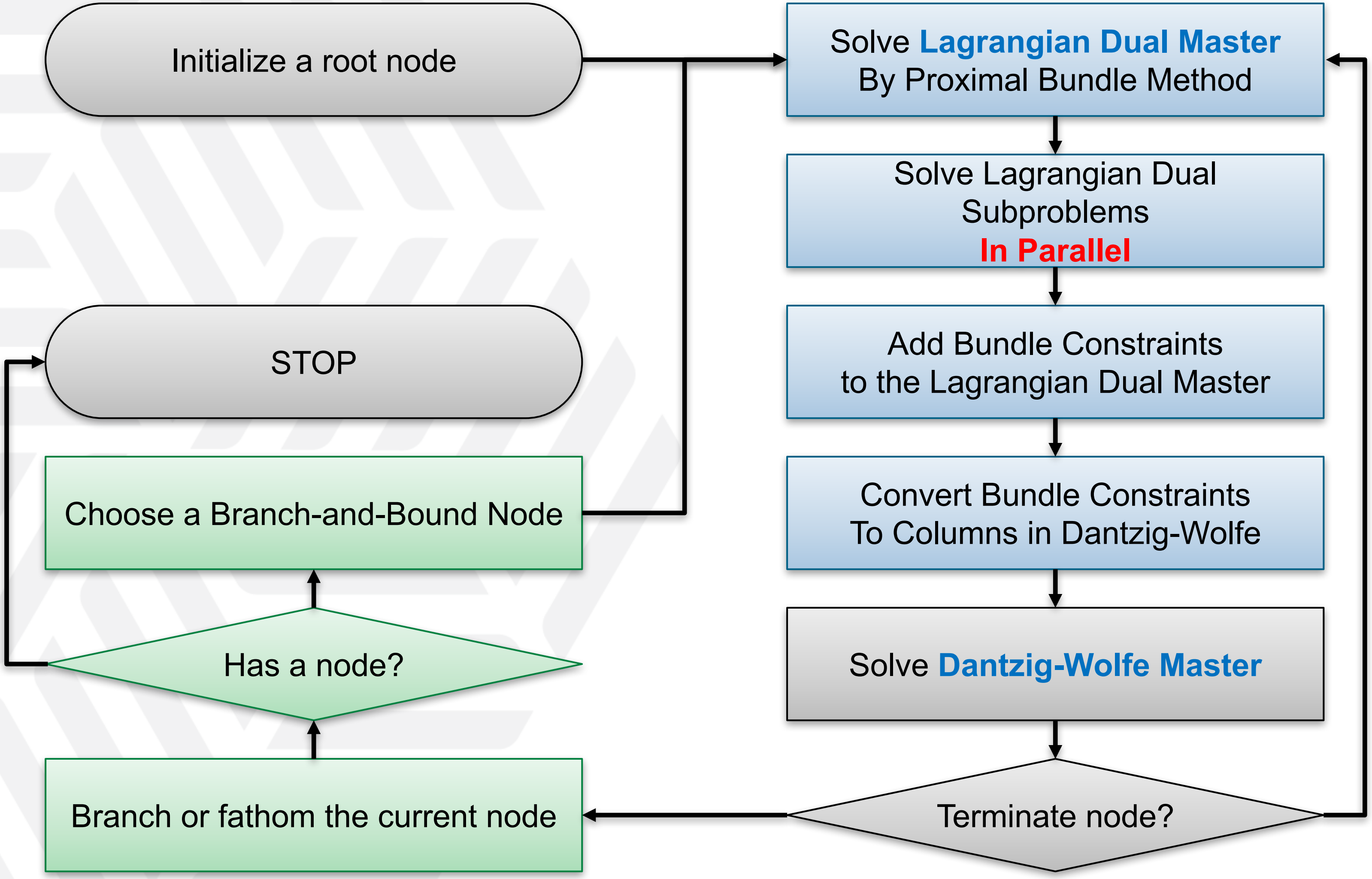
$$\sum_{k\in K} \mathbf{x}_j^k \alpha_j^k \leq \left\lfloor \sum_{k\in K} \mathbf{x}_j^k \hat{\alpha}_j^k \right\rfloor$$

$$\sum_{k\in K} \mathbf{x}_j^k \alpha_j^k \geq \left\lceil \sum_{k\in K} \mathbf{x}_j^k \hat{\alpha}_j^k \right\rceil$$

Adding branching rows

- Also adding branching columns to the DD master problem

Flowchart – Temporal Decomposition



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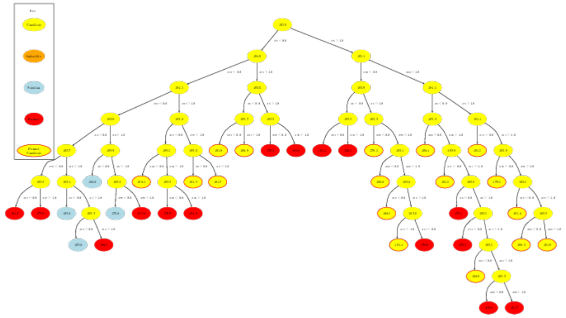
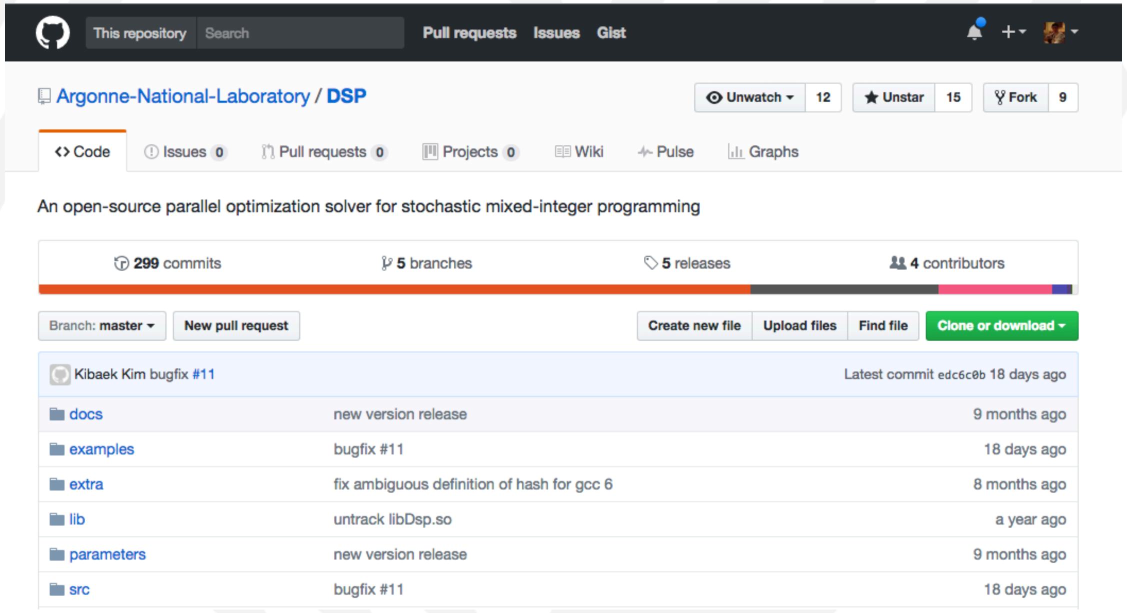
DSP: Scalable Decomposition Solver

- ***Decomposition*** methods for ***Structured Programming***
  - Exploiting block-angular structures
  - **Dantzig-Wolfe decomposition**
    + (Parallel) Branch-and-Bound
  - **Benders decomposition**
  - **Dual decompositio*Parallel***
- Parallel computing via MPI library

$$\begin{array}{llllll} Ax_0 & & & & = b_0 \\ T_1x_0 & +W_1x_1 & & & = b_1 \\ \vdots & & \ddots & & \vdots \\ T_Nx_0 & & & +W_Nx_N & = b_N \end{array}$$

$$\begin{array}{llllll} y_0A & +y_1T_1 & \cdots & +y_NT_N & = \pi_0 \\ & y_1W_1 & & & = \pi_1 \\ & & \ddots & & \vdots \\ & & & +y_NW_N & = \pi_N \end{array}$$





DSP reads models from Julia

$$\min \left\{ -1.5\,x_1 - 4\,x_2 + \sum_{s=1}^{3} p_s Q(x_1, x_2, \xi_1^s, \xi_2^s) : x_1, x_2 \in \{0, \dots, 5\} \right\},$$

where

$$\begin{aligned} Q(x_1, x_2, \xi_1^s, \xi_2^s) = \min_{y_1, y_2, y_3, y_4} \quad & -16y_1 + 19y_2 + 23y_3 + 28y_4 \\ \text{s.t.} \quad & 2y_1 + 3y_2 + 4y_3 + 5y_4 \le \xi_1^s - x_1 \\ & 6y_1 + y_2 + 3y_3 + 2y_4 \le \xi_2^s - x_2 \\ & y_1, y_2, y_3, y_4 \in \{0, 1\} \end{aligned}$$

and $(\xi_1^s, \xi_2^s) \in \{(7,7), (11,11), (13,13)\}$ with probability 1/3.

Only 15 lines of Julia script!

```
using Dsp, MPI # Load packages
MPI.Init() # Initialize MPI
m = Model(3) # Create a Model object with three scenarios
xi = [[7,7] [11,11] [13,13]] # random parameter
@variable(m, 0 <= x[i=1:2] <= 5, Int)
@objective(m, Min, -1.5*x[1]-4*x[2])
for s in 1:3
  q = Model(m, s, 1/3);
  @variable(q, y[j=1:4], Bin)
  @objective(q, Min, -16*y[1]+19*y[2]+23*y[3]+28*y[4])
  @constraint(q, 2*y[1]+3*y[2]+4*y[3]+5*y[4]<=xi[1,s]-x[1])
  @constraint(q, 6*y[1]+1*y[2]+3*y[3]+2*y[4]<=xi[2,s]-x[2])
end
solve(m, solve_type=:Dual, param="myparams.txt")
MPI.Finalize() # Finalize MPI
```

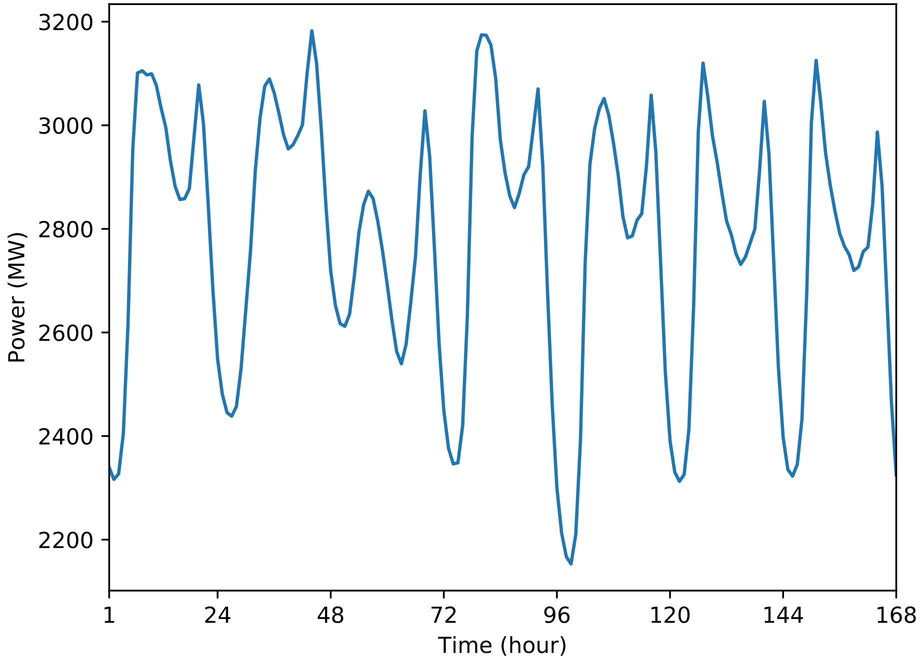
Computational Results – IEEE 118-Bus System

- Implementation
  - Argonne's parallel open-source software: DSP + Coin-ALPS
  - Julia interface for modeling
  - Running on Argonne's Blues cluster (600-node computing node with 16 cores on each)

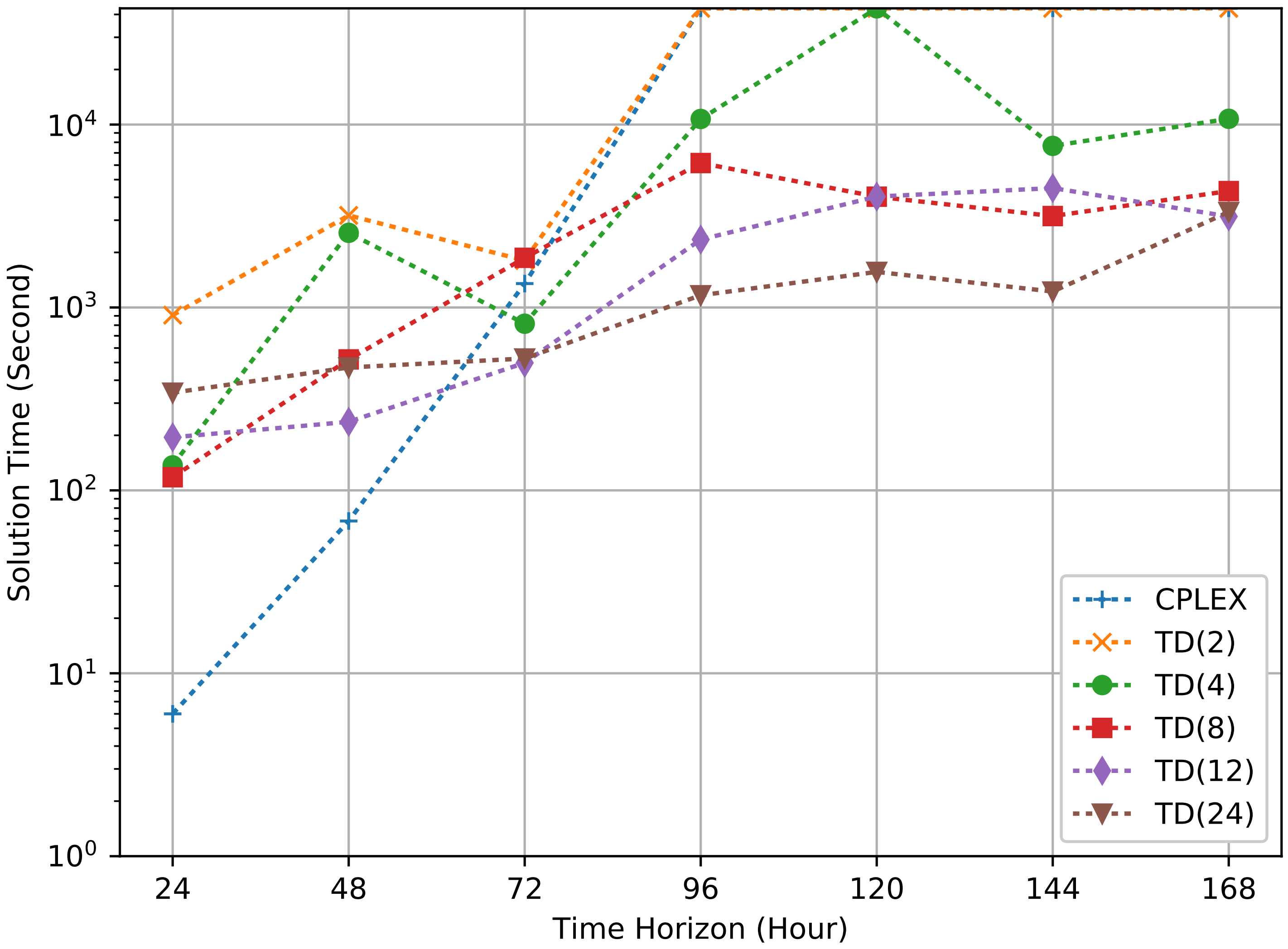
- IEEE 118-Bus System
  - 118 buses, 54 generators, and 186 transmission lines
  - Estimated hourly demand from the PJM system (April 2016)

Table 1: Sizes of IEEE 118-bus system problem instances

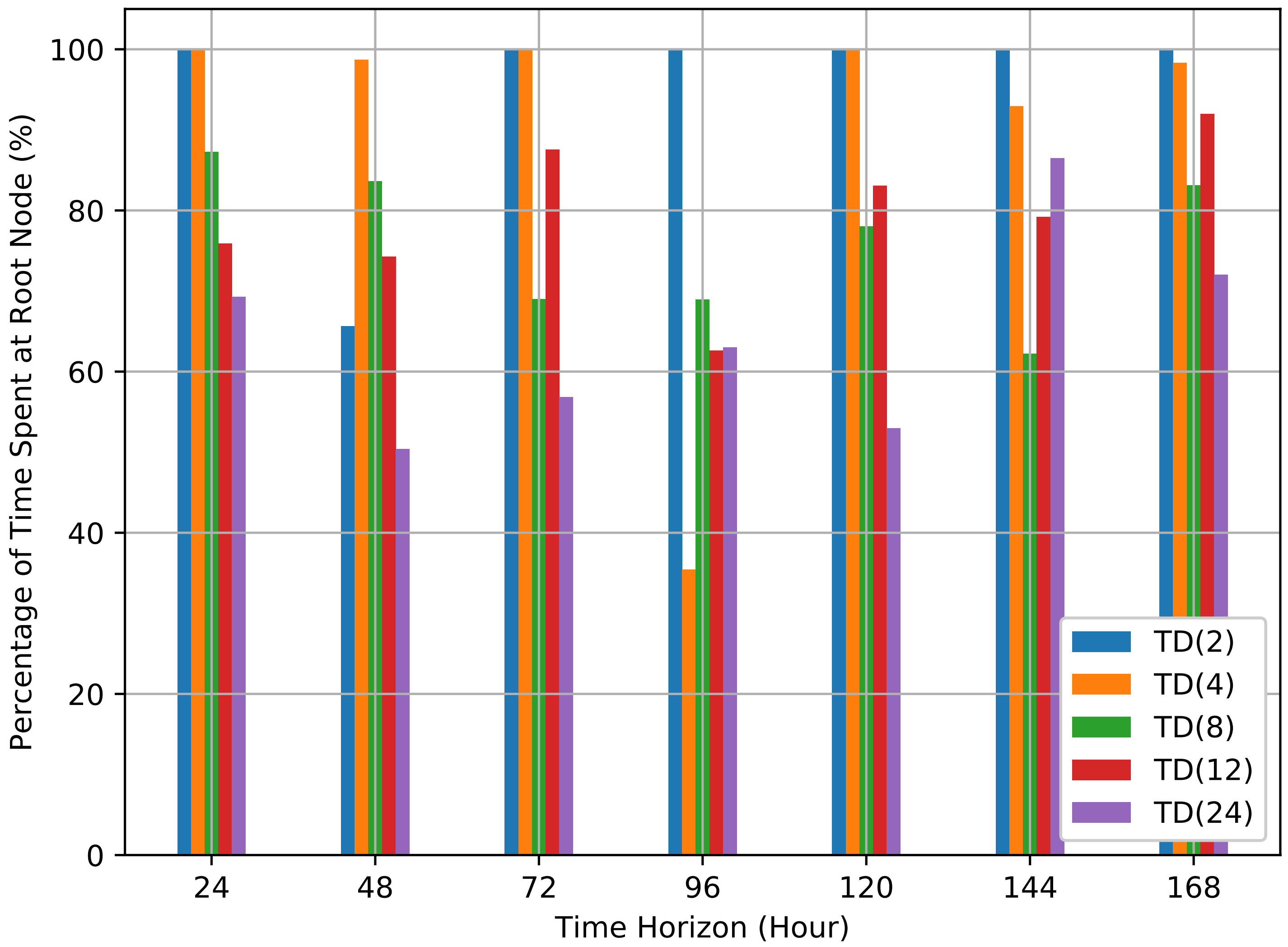
T	# Constraints	# Variables	# Binary
24	19765	18960	1296
48	40070	37920	2592
72	60398	56880	3888
96	80726	75840	5184
120	101054	94800	6480
144	121382	113760	7776
168	141710	132720	9072



Solution Time Benchmark – Log Scale



Computational Results - Branch-and-Bound



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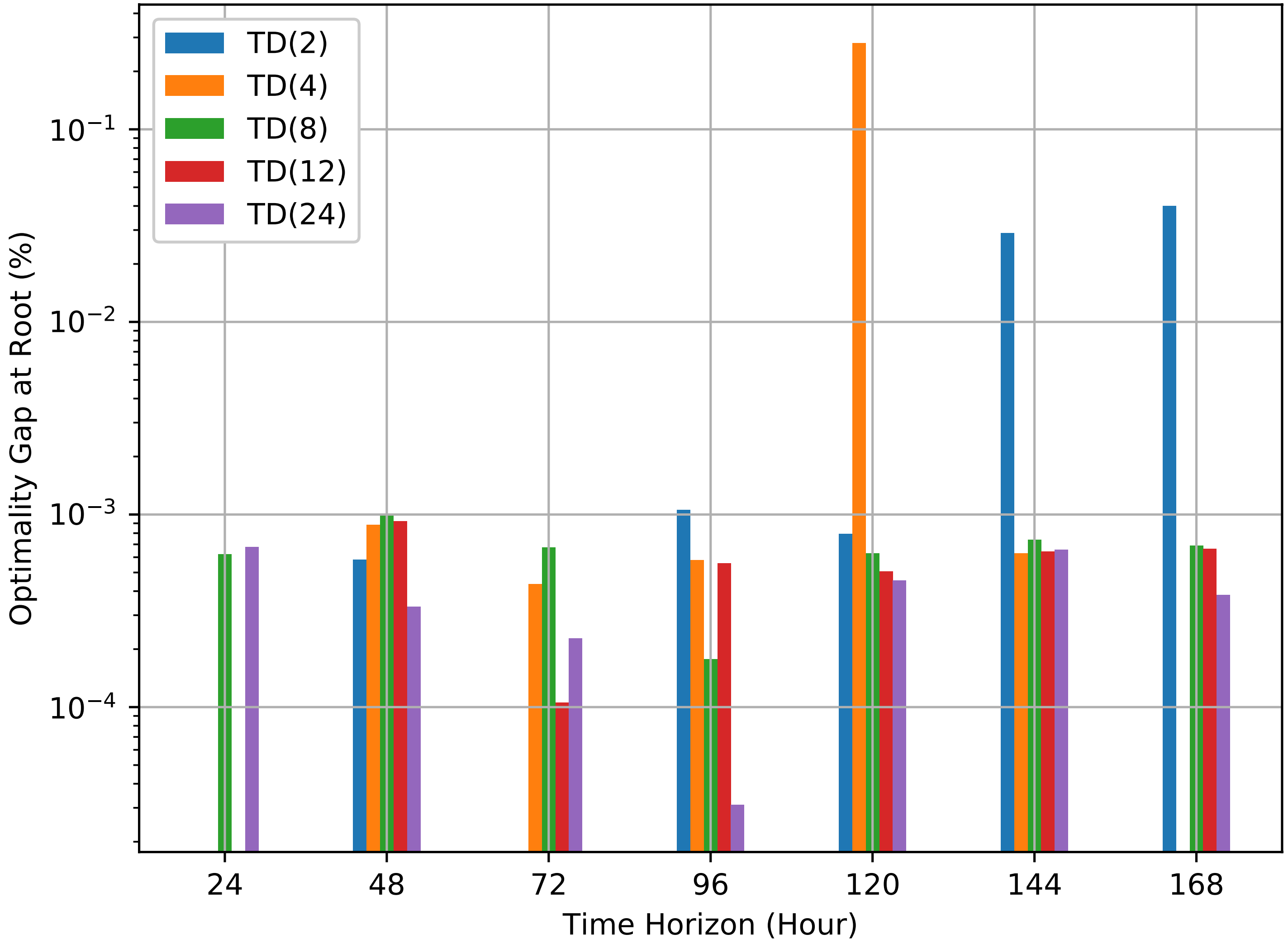
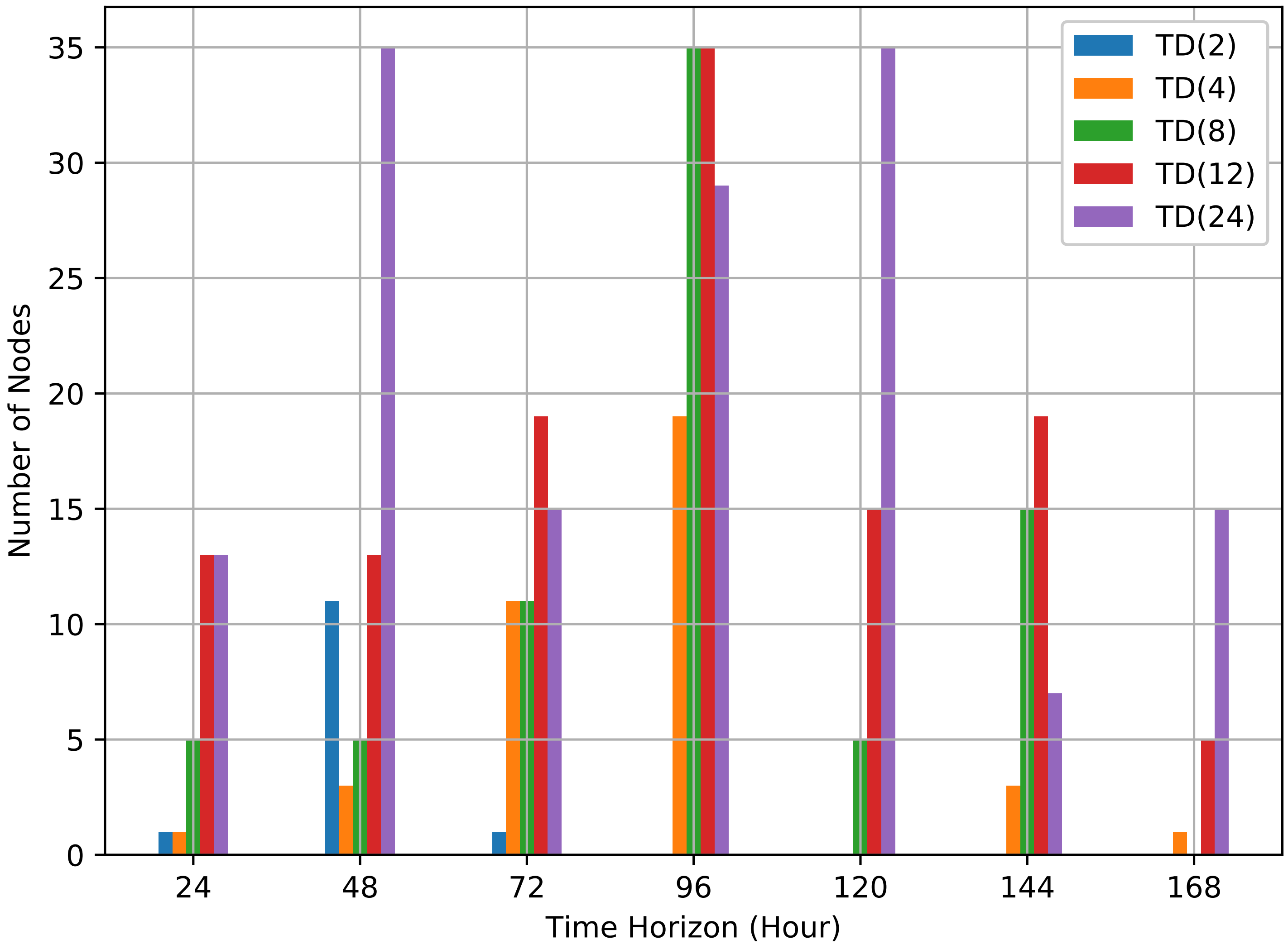


Computational Results - Branch-and-Bound



Future Work

- Algorithms
  - Integrating with generic MIP solution
  - Inexact subproblem solutions
  - Primal cutting planes
  - Primal heuristics
  - Further Parallelization
    - Master problems
    - Tree search
- Investigating Applications
  - Network decomposition
  - Hybrid decomposition (network, time, scenarios)
  - Other applications