Revisiting MIP Gaps and Pricing in RTO-scale Unit Commitment

Brent Eldridge[1,2] and Richard O'Neill[2]

FERC Software Conference

[1] Department of Environmental Health & Engineering, Johns Hopkins University

[2] Office of Energy Policy and Innovation, FERC

June 2018

Outline

- History
- Previous work
- Three pricing models
- Results
- Conclusion

History of Integer Programming in Electricity Markets

- **1982**: *Optimal Spot Pricing of Electricity* (Caramanis *et al.*, 1982)
- **1989**: *Mixed-integer programming is, “theoretically complicated and computationally cumbersome”* -- *EPRI Report* (GS-6401, 1989)
- **1999**: NYISO implements fixed-block pricing in its initial market design
- **1999**: CPLEX 6.5 is released, implementing “theoretical backlog” of performance improvements (Bixby, 2012)
- **2004**: PJM switches from LR to MIP, $90 M/year production cost savings (Ott, 2010)
- **2007**: Gribik, Hogan and Pope (2007) propose Convex Hull Pricing
- **2011**: MISO implements ELMP based on Convex Hull Pricing

Previous Work

What’s the trouble with LMP, anyway?

Price deviations in alternative near-optimal unit commitment solutions

Johnson *et al.* (1997)

- Near-optimal solutions using LR
- Resource profits vary due to changes in prices
- Corresponds to wealth transfers between consumers and generators
- Argues against centralized unit commitment

Sioshansi *et al.* (2008)

- Near-optimal solutions within the MIP gap
- Replicates Johnson *et al.*'s results
- Benefits of MIP
  - Better consistency, but imperfect
  - Lower cost solutions
- Addition of make-whole payments helps mitigate wealth transfers

Pricing Models

- Fixed Model
- Approximate Convex Hull
- Approximate Restricted Convex Hull

Fixed Pricing Model (LMP)

- Standard formulation (O'Neill, 2005), used by Sioshansi *et al.*
- Commitment status w_{gt} is fixed at its optimal value
- Set $\mathcal{Y}_g$ contains all private constraints, except $w_{gt} \in \{0,1\}$
  - Output limits
  - Min up/down time
  - Ramp rates
  - Startup/shutdown logic
- Piecewise linear cost function $C_g(p_{gt})$ and startup cost F_g

$$
\begin{aligned}
\min \quad & \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} \left(C_g(p_{gt}) + F_g z_{gt} \right) \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_{gt} = D_t \\
& (p_{gt}, w_{gt}, y_{gt}, z_{gt}) \in \mathcal{Y}_g \\
& w_{gt} = w_{gt}^*
\end{aligned}
$$

Approximate Convex Hull (aCHP)

- Full CHP is impractical for 24 hour poblem
  - aCHP is exact approximation if ramp rates aren't binding (Hua & Baldick, 2017)
- Cost function $\bar{C}_g(p_{gt})$ is made tighter for PWL cost curves
- All binaries w_{gt} are relaxed

$$
\begin{aligned}
\min \quad & \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} \left(\bar{C}_g(p_{gt}) + F_g z_{gt} \right) \\
\text{s.t.} \quad & \sum_{g \in \mathcal{G}} p_{gt} = D_t \\
& (p_{gt}, w_{gt}, y_{gt}, z_{gt}) \in \mathcal{Y}_g \\
& 0 \leq w_{gt} \leq 1
\end{aligned}
$$

Approximate Restricted CHP (arCHP)

- Relaxes only the set of dispatched generators
  - Variant: relax only hours that gen is dispatched, not tested
- All other aspects same as aCHP model

$$\min \quad \sum_{t \in \mathcal{T}} \sum_{g \in \mathcal{G}} \left(\bar{C}_g(p_{gt}) + F_g z_{gt} \right)$$

$$\text{s.t.} \quad \sum_{g \in \mathcal{G}} p_{gt} = D_t$$

$$(p_{gt}, w_{gt}, y_{gt}, z_{gt}) \in \mathcal{Y}_g$$

$$0 \leq w_{gt} \leq 1, \quad \forall g \in \mathcal{G}^*$$

$$w_{gt} = 0, \quad \forall g \in \mathcal{G} \setminus \mathcal{G}^*$$

Convex Hull Approximation
Cost Function Reformulation

Homogeneous of order k:

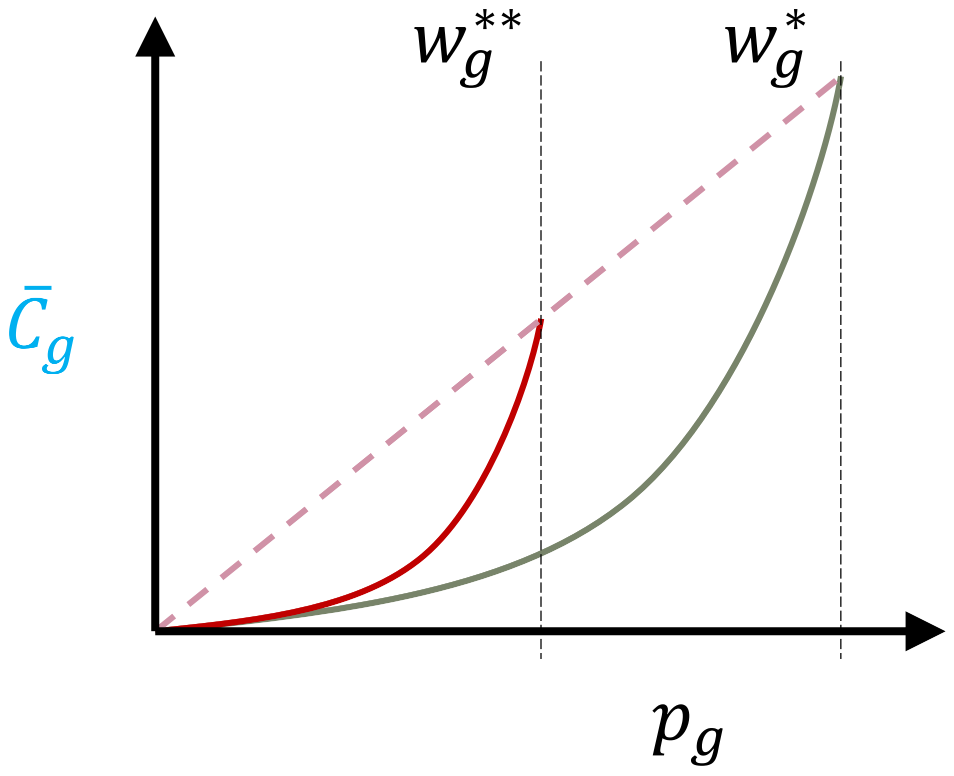
$$f(\alpha x) = \alpha^k f(x)$$

Additional constraints:

$$\bar{C}_g(x) = \sum_{\ell=1}^{L} \widehat{MC}_{g\ell} x_{g\ell}$$

$$\sum_{\ell=1}^{L} x_{g\ell} = p_g$$

$$0 \leq x_{g\ell} \leq w_g \, \Delta p_g$$

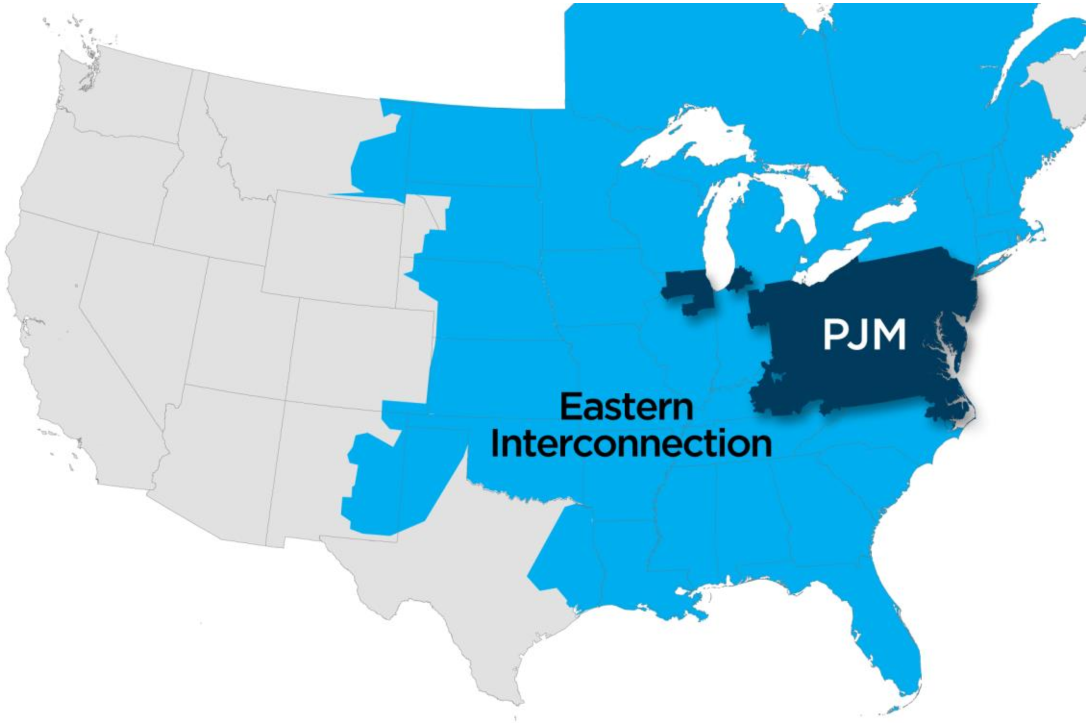


Results

How would different pricing models affect this *inter-solution* price variability?

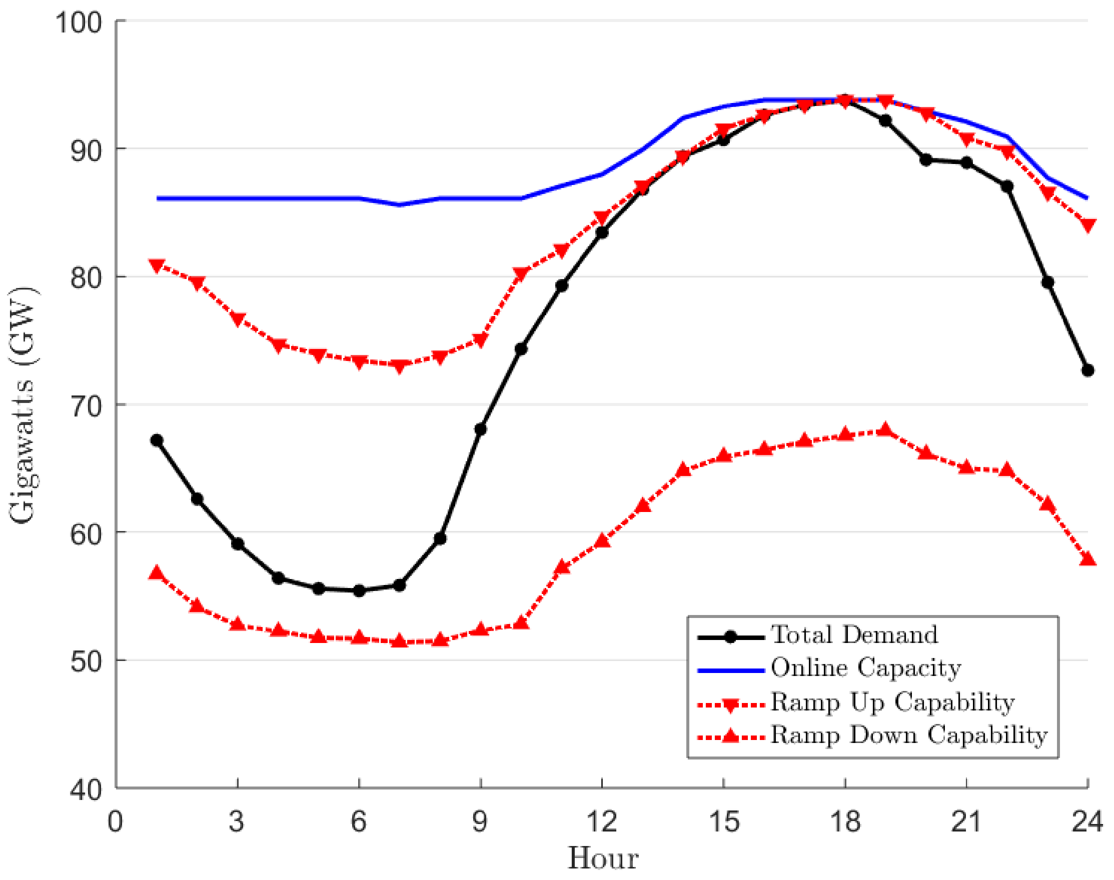
RTO-Scale Test Case (based on PJM)

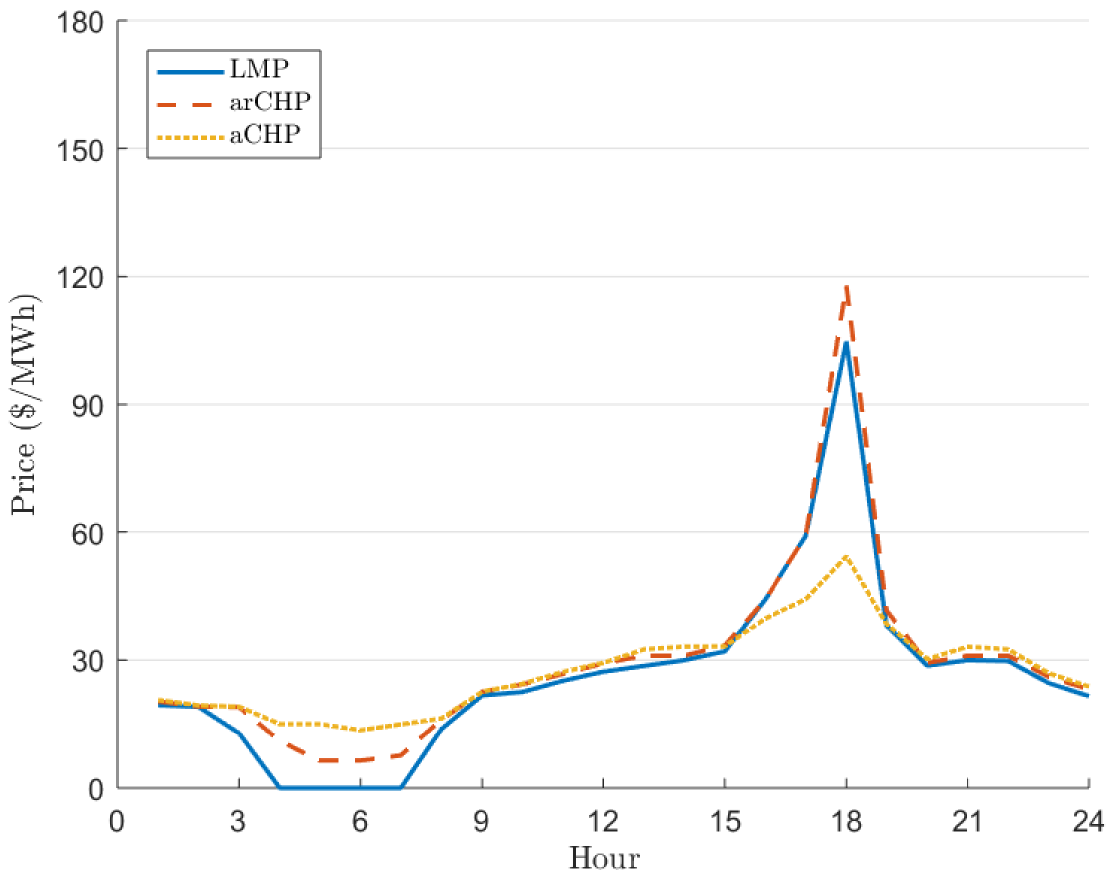
- 24-hour day-ahead unit commitment
- Includes:
  - Piecewise linear generator offers with startup and no-load costs
  - Generator min/max output constraints
  - Min uptime/downtime constraints
  - Ramp rate constraints
  - Fixed demand
- Excludes transmission and reserves
- 293,233 constraints
- 121,321 variables
- 24,264 binary variables



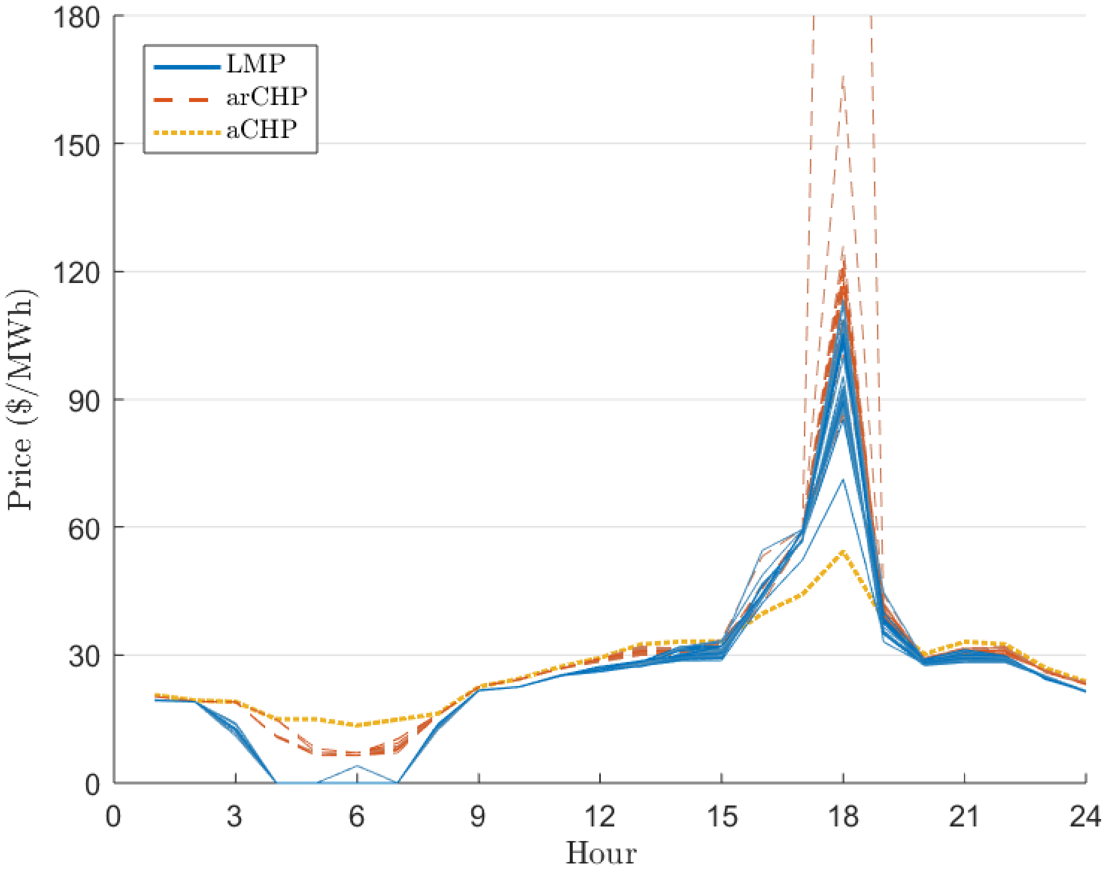
Picture source: http://www.pjm.com/about-pjm/value-proposition.aspx

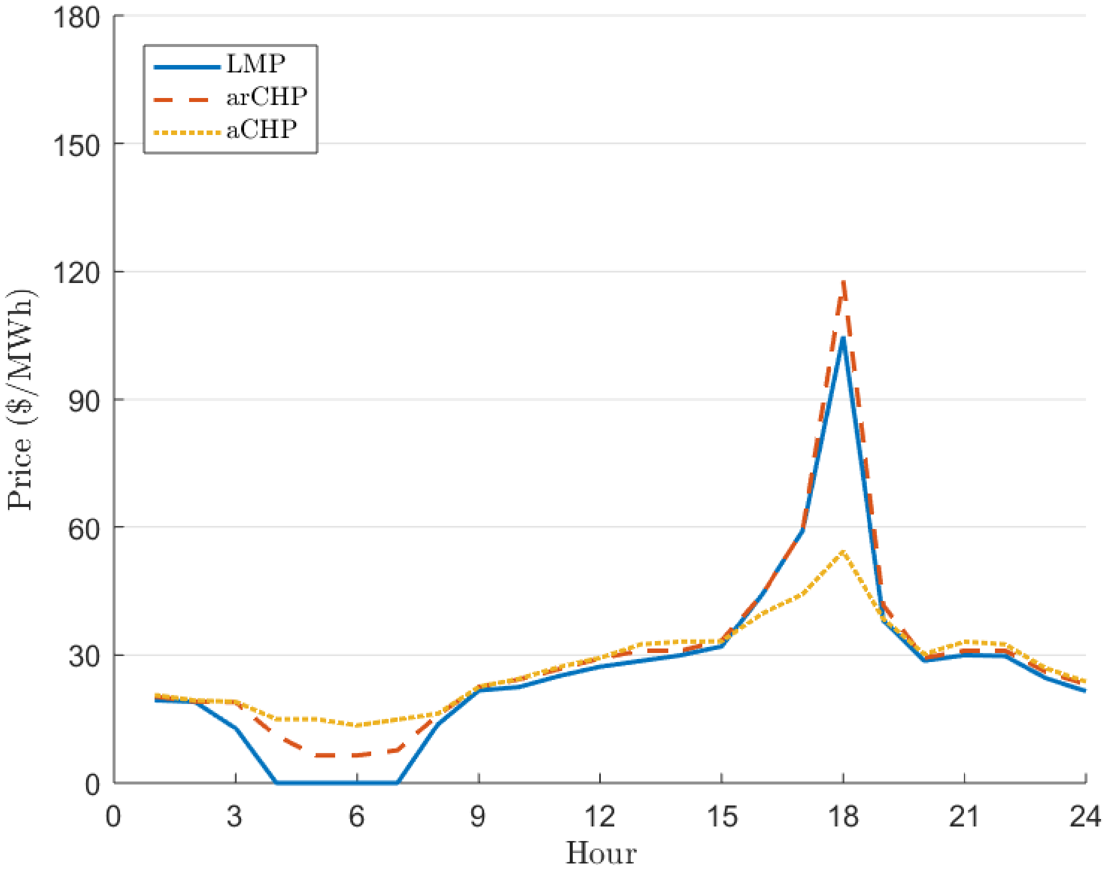
Optimal Solution and Prices





Price deviations: near-optimal vs. optimal





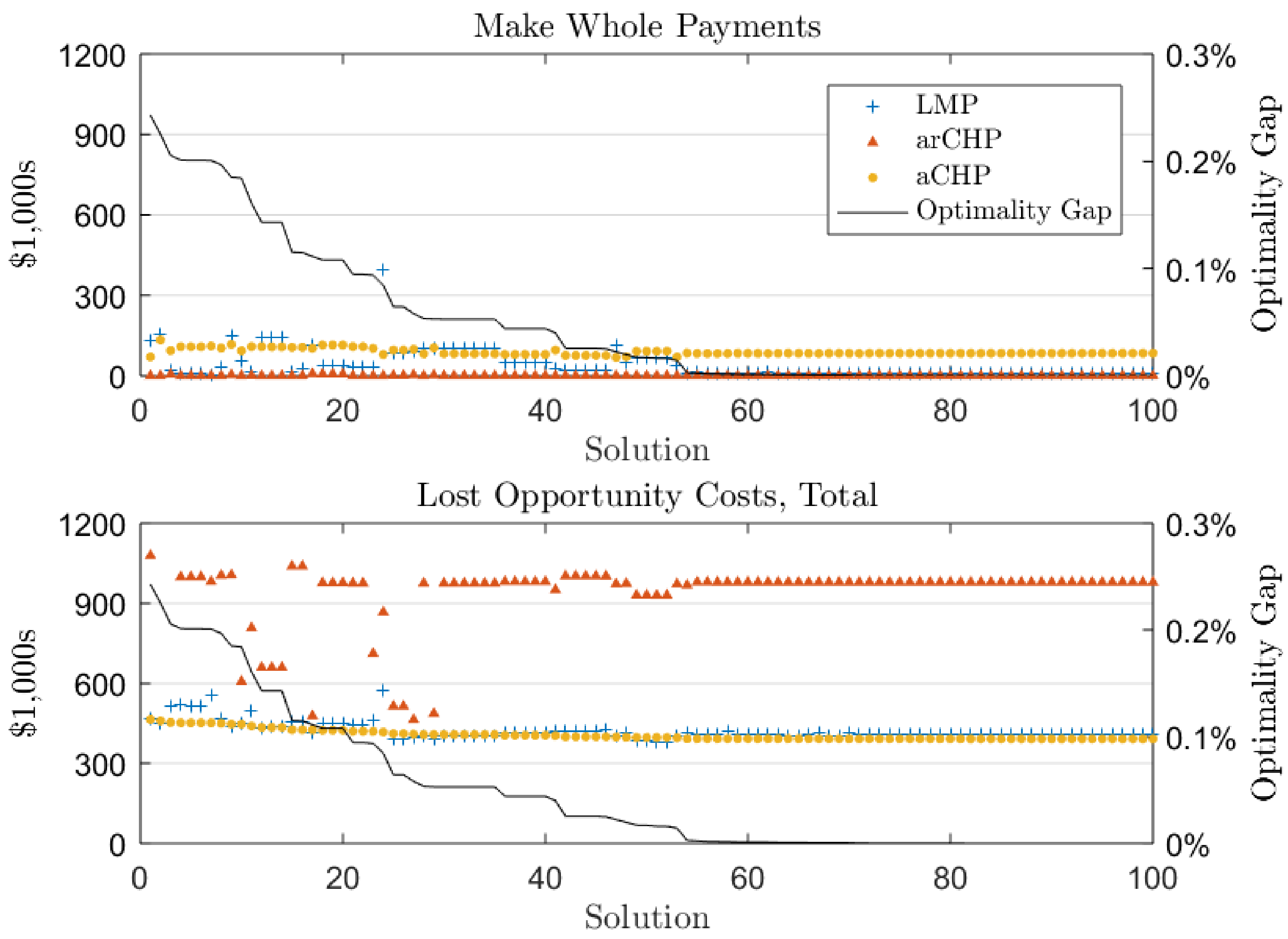
Incentive Compatibility Measures

- ***Make-whole payments* (MWPs)**: amount to ensure bid cost recovery
  - Standard practice, paid to generators in all ISOs
  - Only paid to on-line generators
- ***Lost Opportunity Costs* (LOCs)**: profitability difference of socially optimal and privately optimal schedules
  - Important distinction – measurement of lost opportunity costs does not imply any particular side-payment policy
  - Represents self scheduling incentives and “trust” in the market
  - Creates need for incentive corrections (payments, deviation penalties, etc.)
  - Possible whether generator is on-line or off-line

Make-whole payments & lost opportunity costs

LOC >> MWP, regardless of pricing model

- MWP is a lower bound to (a component of) LOC



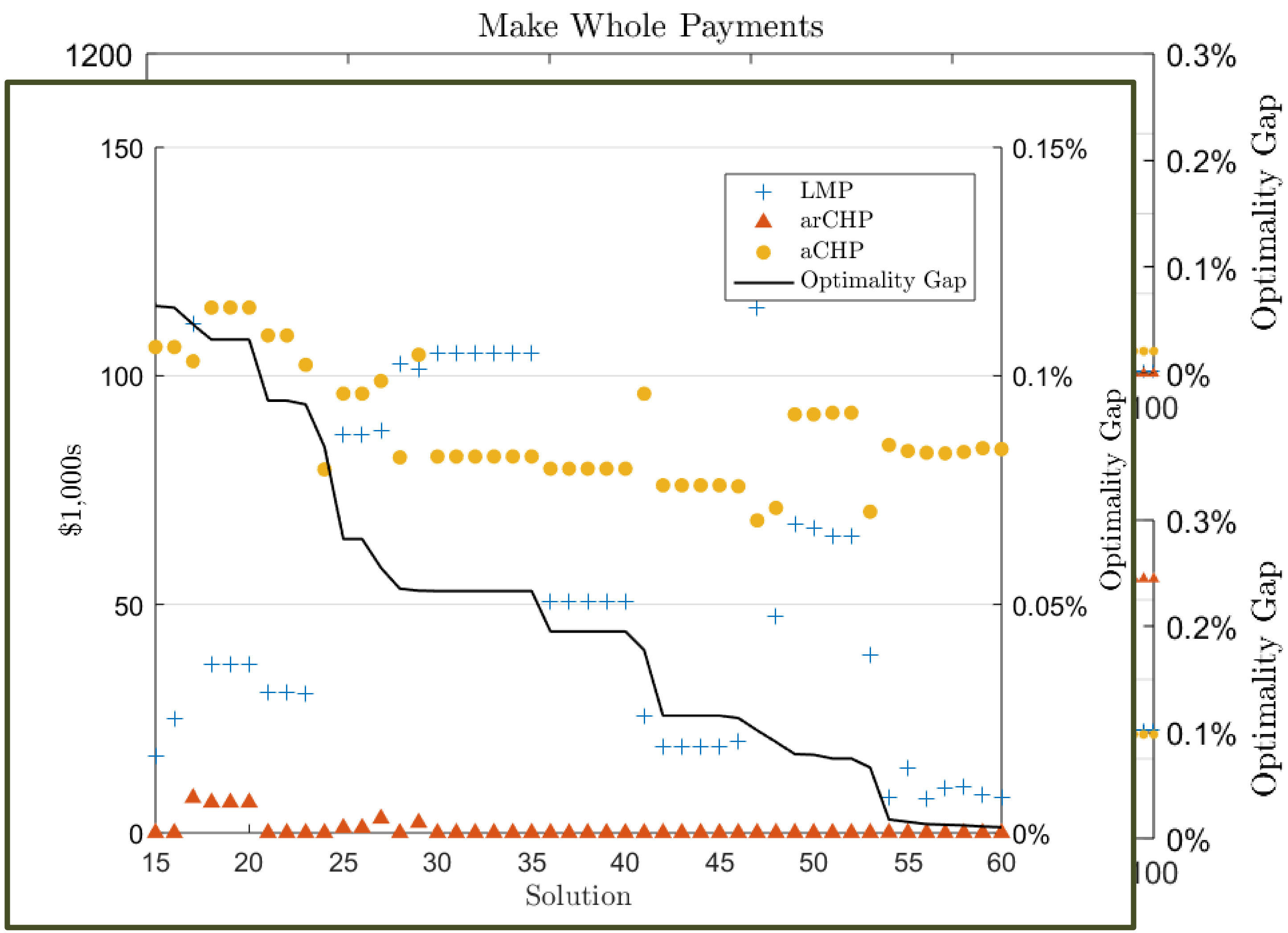
Make-whole payments & lost opportunity costs

LOC >> MWP, regardless of pricing model

- MWP is a lower bound to (a component of) LOC

No relation for MWPs in aCHP compared to LMP

High peak price in restricted model (arCHP) mostly eliminates MWPs



Lost opportunity costs: on-line & off-line units

Approximate CHP (aCHP) distributes more LOC to online units, less LOC to offline units

- Important: generator may be in the optimal solution but not others

LMP has lower online LOC than arCHP, which is odd

- Poor approximation?

Wealth transfers

$$\Delta\text{EnergyPayment}_s + \Delta\text{MWP}_s = \Delta\text{GenProfit}_s + \text{MIPGap}_s$$

Where:

$$\Delta\text{EnergyPayment} = \sum_t (\text{price}_t^s - \text{price}_t^*) \times D_t$$

$$\Delta\text{MWP} = \text{MWP}^s - \text{MWP}^*$$

$$\Delta\text{GenProfits} = \sum_g \left(\pi_g^s - \pi_g^*\right)$$

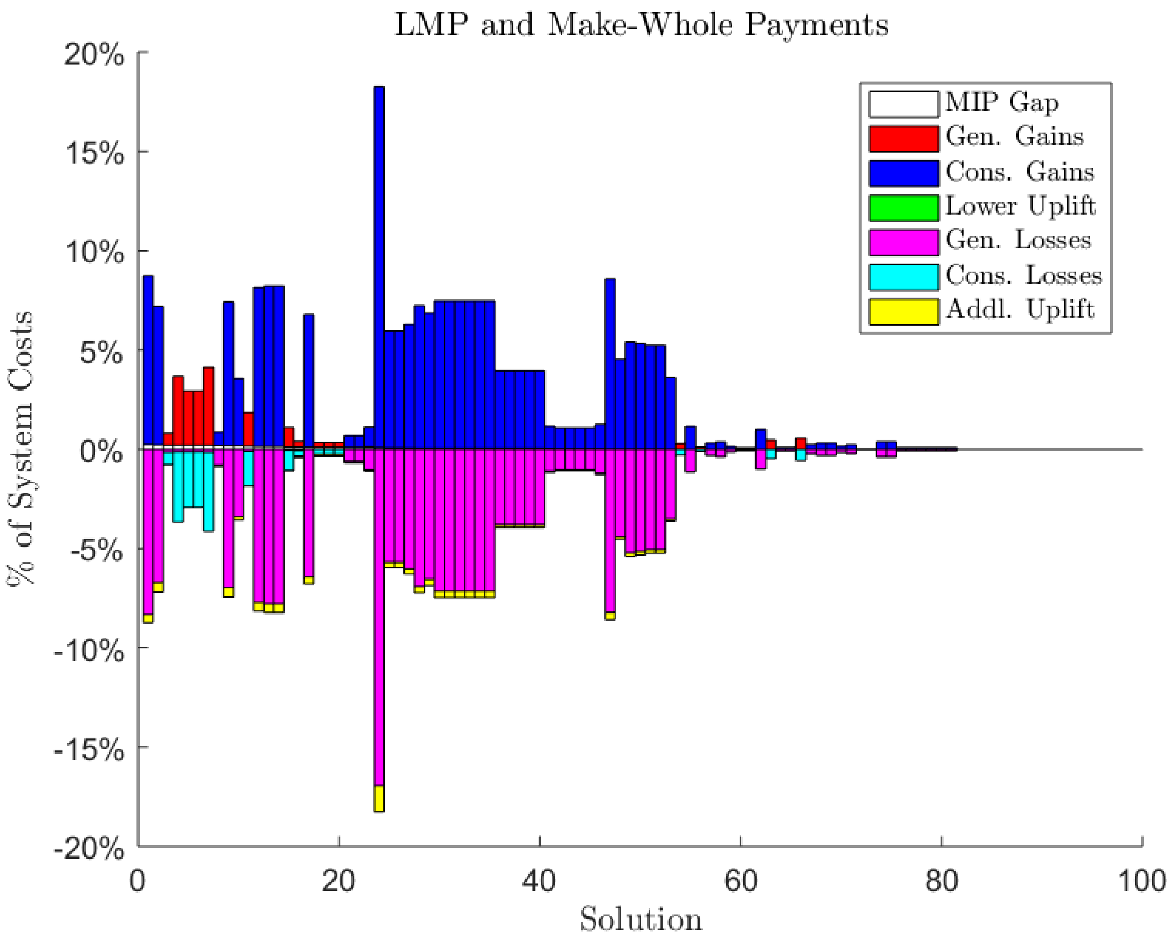
Wealth Transfers: LMP

Compared to payments in the optimal solution

Based on LMPs and make-whole payments

Replicates Johnson *et al.* and Sioshansi *et al.* results

24 solutions with transfers more than 5% of the system cost



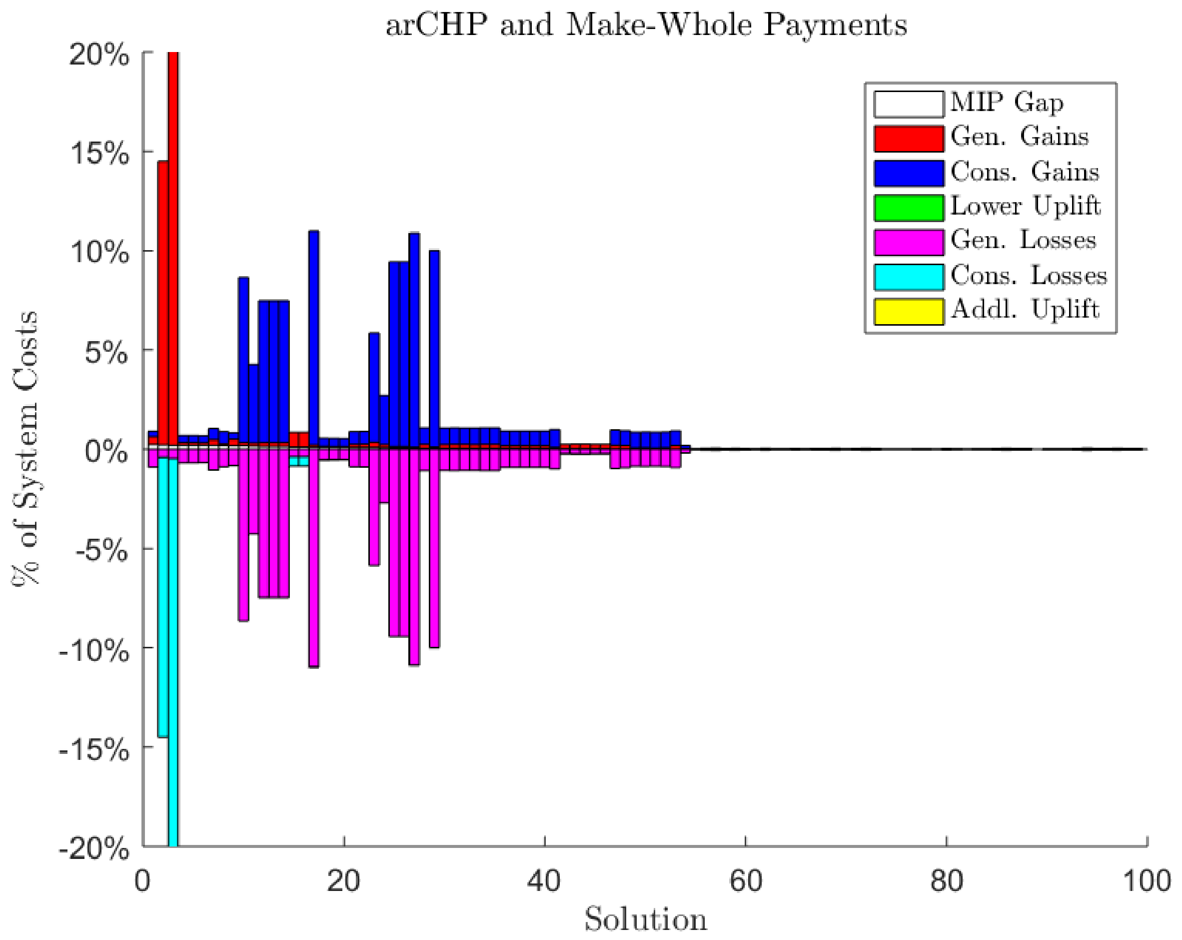
Wealth Transfers: arCHP

Compared to payments in the optimal solution

Based on arCHPs and make-whole payments

12 solutions with transfers more than 5% of system cost

Max transfer is 118% of the system cost (3rd solution)

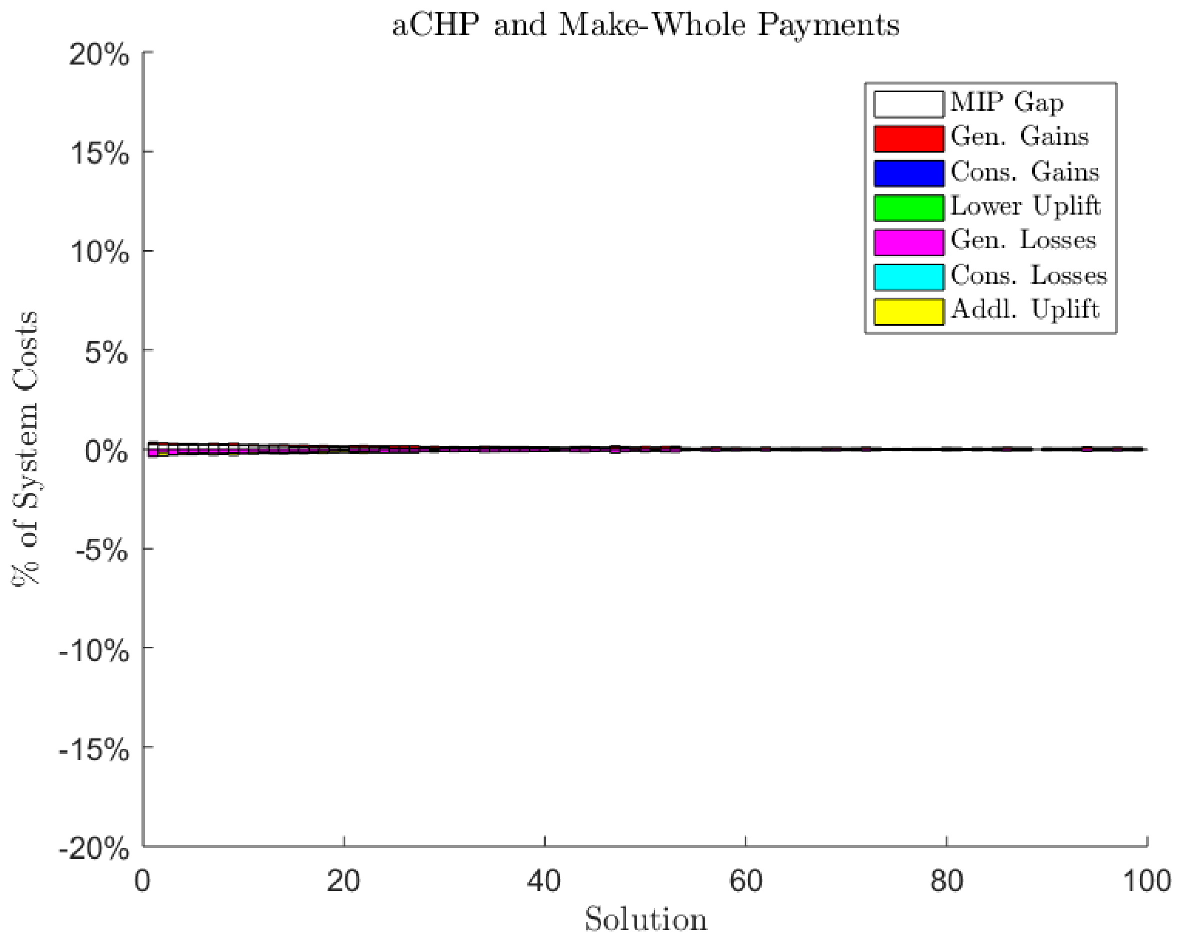


Wealth Transfers: aCHP

Compared to payments in the optimal solution

Based on aCHPs and make-whole payments

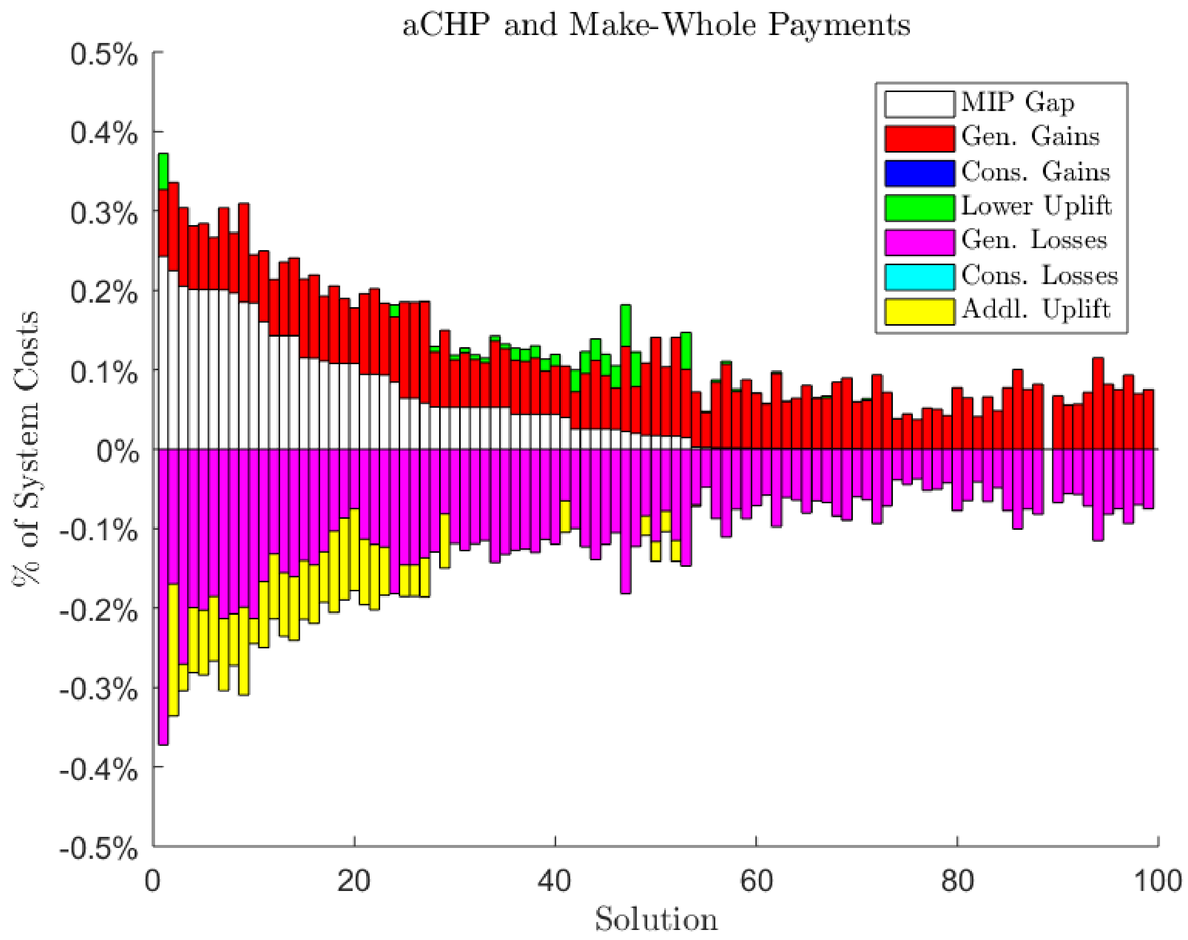
Comparatively few transfers between alternative solutions



Wealth Transfers: aCHP (zoomed in)

Unlike other methods, wealth transfers are primarily between generators

Small size indicates level of indifference between alternative solutions



Wealth transfers in the first 50 solutions, solution price to optimal solution price

Price:	LMP	arCHP	aCHP
Average (% system cost)	4.5%	5.1%	0.19%
Maximum (% system cost)	18%	118%	0.37%
# > 1.0%	42	22	0
# > 2.5%	33	14	0
# > 5.0%	22	12	0
# > 7.5%	6	8	0

Conclusions

Conclusions

- From Sioshansi *et al.* (2008): Unit commitment is a deterministic algorithm, so wealth transfers are likely to persist over days with similar conditions
  - i.e., transfers *do not* cancel out over time
  - Possible gaming opportunities and rent seeking behavior
- Convex hull pricing removes this instability
  - No discontinuities → simpler economic bidding incentives
  - Indifference among participants who are only in *some* of the near-optimal solutions
  - High LOC represents willingness to be a price taker (to self-schedule)
  - Need to be addressed: Day ahead and real-time market convergence and incentives to follow dispatch (payments or penalties?)
- Paradoxically, results have little to do with lowering uplift payments
  - Paying LOC may be undesirable due to strategic bidding

Perfect theory of ~~forms~~
optimal solutions

-- or --

Empiricism, approximation,
and large-scale problems

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Caramanis, Michael C., Roger E. Bohn, and Fred C. Schweppe. "Optimal spot pricing: Practice and theory." *IEEE Transactions on Power Apparatus and Systems* 9 (1982): 3234-3245.

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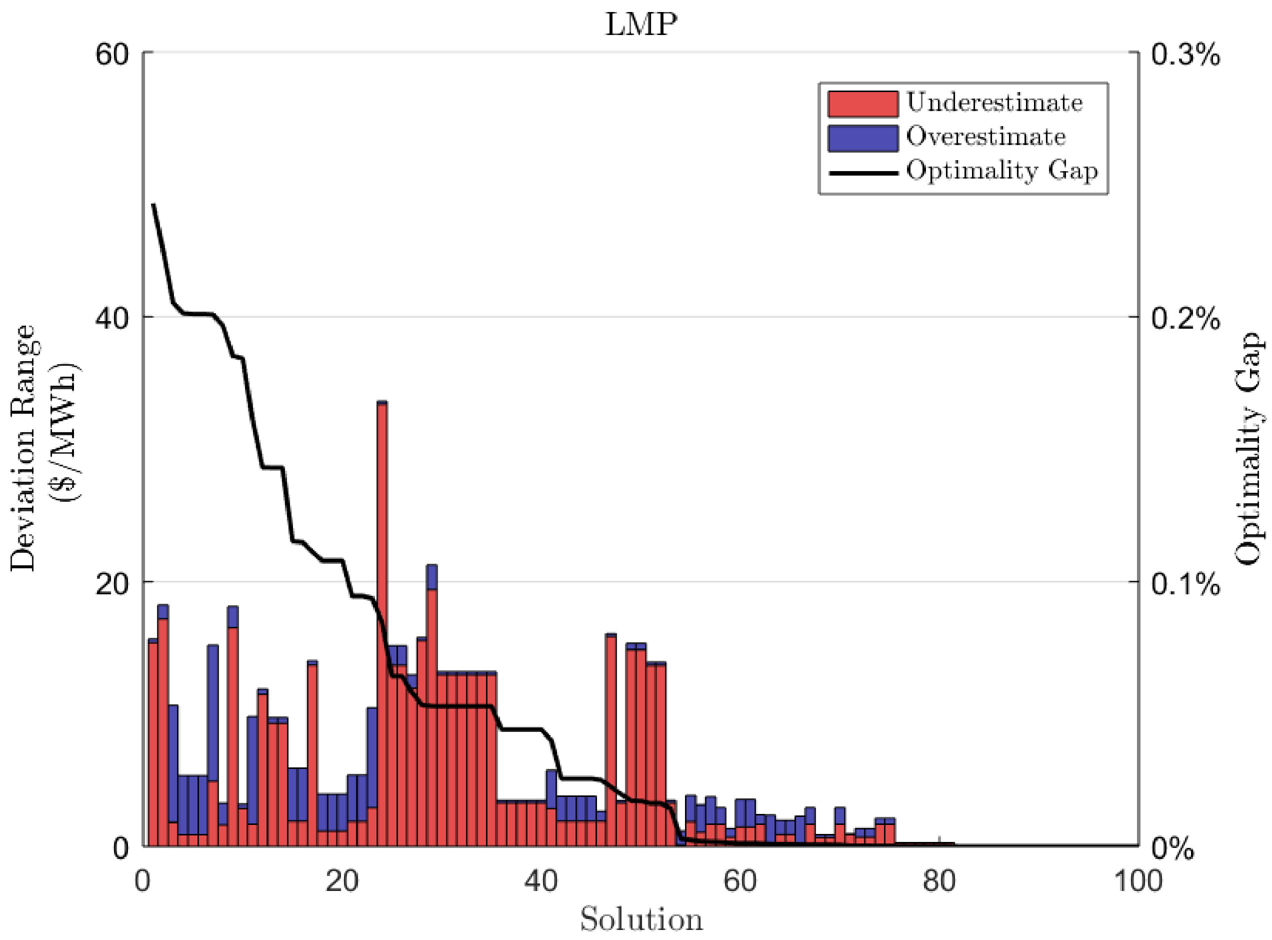
Questions?

Price Deviations: LMP

$$O_s = \max_t \left(LMP_t^s - LMP_t^*\right)$$
$$U_s = \min_t \left(LMP_t^s - LMP_t^*\right)$$

Replicates Johnson *et al.* and Sioshansi *et al.* results

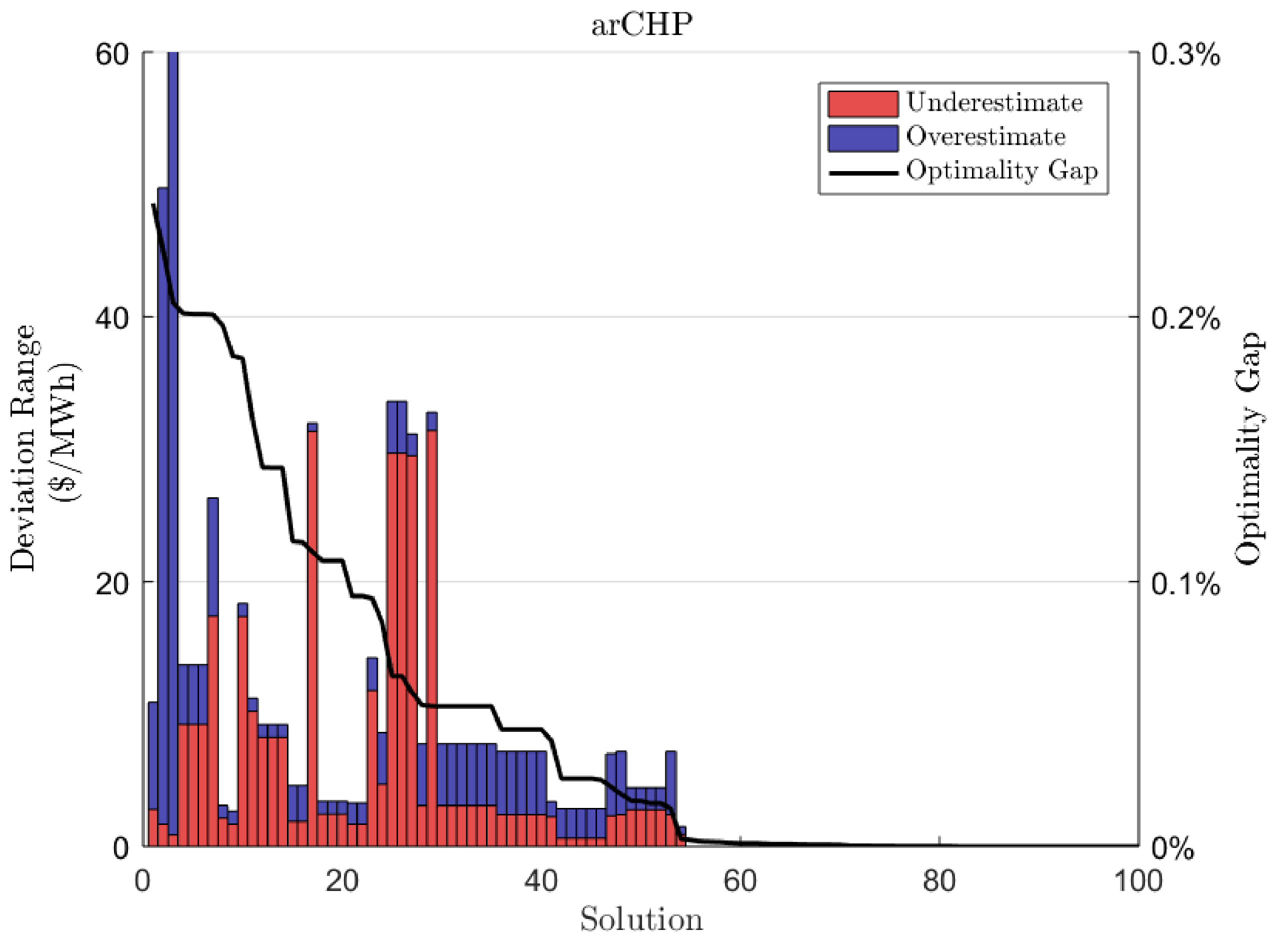


Price Deviations: arCHP

$$O_s = \max_t \left(arCHP_t^s - arCHP_t^*\right)$$
$$U_s = \min_t \left(arCHP_t^s - arCHP_t^*\right)$$

Similar to LMP, maybe smaller in most solutions

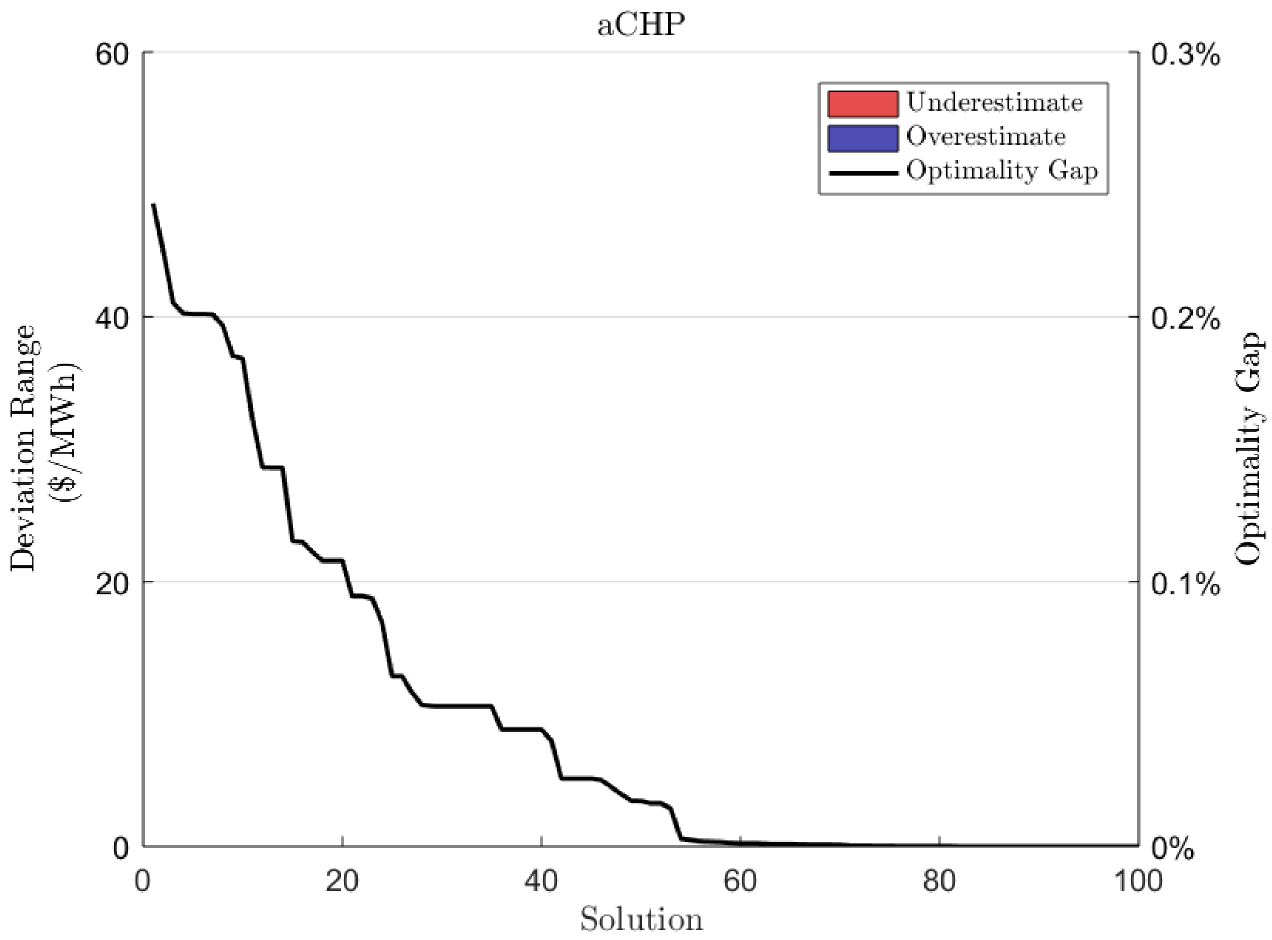
($378 deviation in 3rd solution)



Price Deviations: aCHP

$$O_s = \max_t \left(aCHP_t^s - aCHP_t^*\right)$$
$$U_s = \min_t \left(aCHP_t^s - aCHP_t^*\right)$$

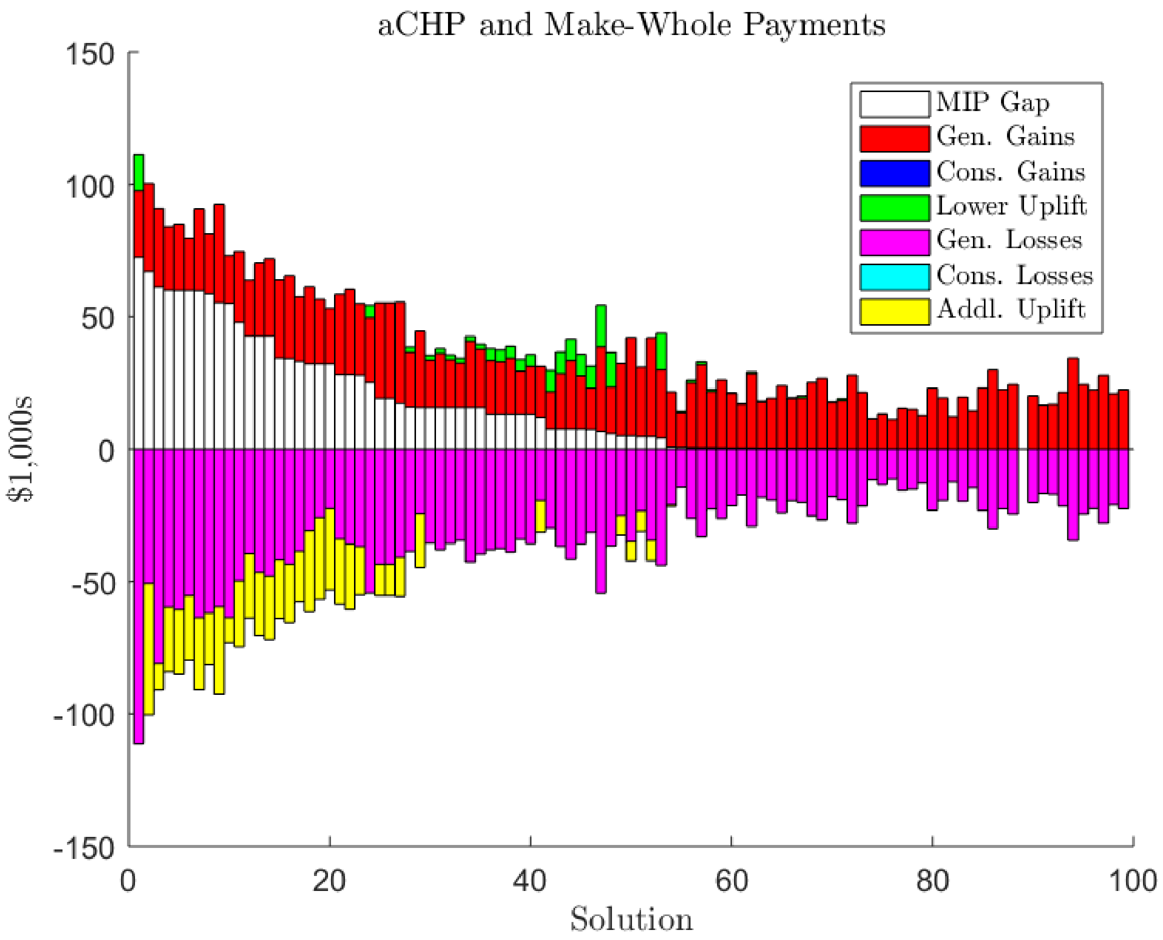
All units are relaxed for aCHP, so no price deviations



Wealth Transfers: aCHP (in Dollars)

Unlike other methods, wealth transfers are primarily between generators

Small size indicates level of indifference between alternative solutions



Johnson *et al.* (1997): Alternative near-optimal solutions using LaGrangian relaxation

"However, the ***aggregate resource profits vary by up to 6% percent due to differences in the price vectors*** corresponding to the different solutions. Thus, while all the solutions are equally efficient they have different equity implications since the profit variability ***corresponds to welfare transfer between generators and consumers***."

TABLE 1:

SIMULATION RESULTS SUMMARY

Run	TOTALS (K$)		
	Cost	Payment	Profits
1	20,306.94	30,176.44	9,869.51
2	20,310.31	30,275.33	9,965.01
3	20,305.80	30,303.42	9,997.62
4	20,307.91	30,237.94	9,930.04
5	20,311.07	30,255.59	9,944.51
6	20,318.74	30,509.02	10,190.28
7	20,321.90	30,238.11	9,916.21
8	20,319.36	30,438.41	10,119.05
9	20,321.70	29,929.26	9,607.56
10	20,305.80	30,283.39	9,977.59
11	20,307.90	30,293.26	9,985.36
12	20,310.30	30,307.03	9,996.73
average	20,312.31	30,270.60	9,958.29
std	6.28	140.08	140.61
max	20,321.90	30,509.02	10,190.28
min	20,305.80	29,929.26	9,607.56
range	16.10	579.77	582.72
range/avg	0.08%	1.92%	5.85%
std/mean	0.03%	0.46%	1.41%

Sioshansi *et al.* (2008): Alternative near-optimal solutions within the MIP gap

- Replicates price volatility in B&B tree
- Benefits of MIP:
  - Lower cost solutions
  - Pricing is more consistent than Lagrangian Relaxation
- Make-whole payments mitigated generator profitability variances

Solution	Mean	Max	Min
1	2.27%	7.13%	-2.36%
2	2.51%	9.87%	-4.40%
3	3.65%	10.22%	-2.08%
4	3.17%	10.22%	-3.44%
5	3.05%	10.22%	-3.44%
6	3.90%	10.40%	-5.08%
7	3.90%	10.40%	-5.08%
8	3.81%	10.22%	-2.08%
9	3.31%	10.22%	-5.08%
10	3.60%	10.22%	-5.08%
11	3.48%	10.22%	-5.08%
12	3.38%	10.22%	-5.08%
13	1.29%	4.60%	-4.40%
14	1.00%	4.60%	-4.40%
15	1.05%	4.60%	-4.40%
16	1.00%	4.60%	-4.40%
17	-0.19%	5.03%	-7.19%
18	1.03%	5.03%	-7.19%
19	0.77%	5.03%	-4.40%

Solution	Mean	Max	Min
20	0.89%	5.03%	-2.08%
21	0.89%	5.03%	-2.08%
22	0.14%	5.03%	-3.03%
23	1.18%	5.03%	-1.83%
24	1.21%	5.03%	-1.06%
25	1.33%	5.03%	-1.83%
26	1.41%	5.03%	-0.47%
27	1.10%	4.60%	-1.83%
28	1.20%	4.60%	-0.47%
29	1.20%	4.60%	-0.47%
30	0.04%	1.43%	-0.47%
31	-0.02%	0.00%	-0.47%
32	0.17%	3.13%	-0.47%
33	0.11%	3.13%	-0.47%
34	1.35%	4.60%	0.00%
35	1.29%	4.60%	0.00%
36	1.23%	4.60%	-1.06%
37	-0.13%	0.00%	-3.03%
38	0.06%	1.43%	0.00%

Sioshansi *et al.* (2008): Alternative near-optimal solutions within the MIP gap

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18	1.03%	5.03%	-7.19%
19	0.77%	5.03%	-4.40%

So[illegible]			
20			
21			
22			
23			
24			
25			
26	1.41%	5.03%	-0.47%
27	1.10%	4.60%	-1.83%
28	1.2[illegible]	4.60%	-0.47%
29	1.20%	4.60%	-0.47%
30	0.04%	1.43%	-0.47%
31	-0.02%	0.00%	-0.47%
32	0.17%	3.13%	-0.47%
33	0.11%	3.13%	-0.47%
34	1.35%	4.60%	0.00%
35	1.29%	4.60%	0.00%
36	1.23%	4.60%	-1.06%
37	-0.13%	0.00%	-3.03%
38	0.06%	1.43%	0.00%

- Large range of price deviations compared to optimal solution
- Nonmonotonic with decreasing MIP gap